NONLINEAR INTERACTION OF ELECTROMAGNETIC WAVES WITH GASEOUS AND SEMICONDUCTOR PLASMAS

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PREFACE

The interaction of electromagnetic waves with the ionized media has attracted the attention of a large number of theoretical as well as experimental investigators because of its applications to many fields of science and technology. Of special interest to us at I.I.T. are the phenomena associated with the passage of high intensity electromagnetic waves, when the complex conductivity tensor of the medium becomes a function of the electric vector of the propagating wave; this dependence causes a number of interesting nonlinear effects, e.g., nonlinear propagation, demodulation, cross-modulation and harmonic generation.

In this thesis, the author has investigated:-

Part I - The optimum conditions for maximizing the power of the generated harmonics in gaseous and semiconductor plasmas

and

Part II - The nonlinear propagation of an amplitude modulated electromagnetic wave in a magnetoplasma.

Part I It is well known that when an ionized medium is subjected to a moderately strong electric field, its response to the field becomes nonlinear. If the applied electric field is alternating, then the nonlinear effects in the plasma lead to the generation of odd harmonic components in the current density (Rosen, 1961). Baird and Coleman (1961) and Murphy (1965), on the basis of elementary approach, have pointed out that in addition to the odd harmonic components in the current density even harmonic components in the current density are also generated when a plasma is subjected to a d.c. electric field and an alternating electric field simultaneously. Chiyoda (1965) and Sodha and Kaw (1966) suggested that even harmonic components in the current density can also be generated if the alternating electric field is applied to an inhomogeneous plasma having gradients of temperature and electron density. An important consequence of the generation of these higher order components in the current density is the generation of electromagnetic waves at these harmonic frequencies when a strong electromagnetic wave propagates in the plasma. To assess the practical possibility of the generation of harmonics in a plasma, Sodha and Kaw (1966) have also investigated the amplitudes of the electric vectors of these higher frequency components in the reflected wave from a semi-infinite plasma-free space interface when an electromagnetic wave is incident normally on the interface from the free space side.

In actual practice, the confinement of plasmas over large dimensions is a highly complex problem and hence, the generation of harmonics in a semi-infinite plasma becomes very difficult to realise experimentally. If one considers a plasma slab of certain thickness Z_0, then the magnitude of the harmonics in the transmitted wave will be zero in both the cases when $Z_0 = 0$ and $Z_0 \rightarrow \infty$ because in the first case there is no nonlinear medium to interact with while in the second case all the waves will be completely absorbed in the medium. This suggests that there is an optimum thickness of the plasma slab for which the magnitude of these harmonics in the transmitted wave, will be maximum. The optimum magnitude of these harmonics from the plasma slab will be enhanced in the presence of an external magnetic field for the case when the wave frequency becomes equal to the electron gyrofrequency; this is due to resonance effects at gyrofrequency.

The author, in the first three chapters of the thesis, has investigated the optimum conditions for the generation of maximum power of second and third harmonics in a plasma in the presence of (1) an external d.c. electric field, (2) the inhomogeneities due to electron density and temperature gradients and (3) an external d.c. magnetic field respectively.

While the gaseous plasmas offer a high conversion efficiency under similar operating conditions because of the lower collision frequency, the semiconductors offer much more ease of operation. Moreover, we can reduce the collision frequency in semiconductors by reducing the temperature of the sample to very low values (Sodha and Srivastava, 1967). Making use of the close analogy of electron conduction in gaseous plasmas and simple model semiconductors, as pointed out by Shockley (1951), Sodha et al. (1968) and Sodha and Gupta (1969) extended the treatment of Sodha and Kaw (1966) to obtain the magnitudes of second and third harmonic components in the reflected and transmitted waves from a simple model semi-conducting slab (viz. a germanium slab) of arbitrary thickness in the presence of d.c. electric field and inhomogeneities in the medium respectively. In Chapters IV and V of the thesis, the author has improved the treatment of the above workers and has obtained the optimum magnitudes of the various relevant parameters for which the magnitudes of the electric vectors of the second and third harmonic components in the reflected and transmitted waves from a homogeneous (in the presence of an external d.c. electric field) and an inhomogeneous semi-conducting slab respectively, are maximum.

The assumption of parabolic energy bands for simple model semiconductors, is not valid for III-V semiconductors which are characterized by low band gap, non-parabolic energy bands and spherical energy surfaces. III-V compounds are high mobility semiconductors and hence, offer the possibility of higher harmonic conversion efficiencies; one of the promising material under this category is indium antimonide. In Chapters VI and VII, the author has investigated the optimum magnitude of the electric vector of the second harmonic component in the reflected and transmitted waves from a homogeneous and inhomogeneous indium antimonide respectively.

The titles and the summaries of the first seven chapters of the thesis are given below:

CHAPTER I OPTIMUM HARMONIC GENERATION IN HOMOGENEOUS PLASMAS

In this chapter, the author has calculated the optimum magnitudes of the various parameters for which the efficiencies of the second and the third harmonic conversion in the reflected and transmitted waves from a plasma slab, are maximum. The expressions for the second and third harmonic components of the current density in a plasma subjected to an external d.c. electric field and an alternating electric field of an electromagnetic wave, are taken to be those derived by Sodha and Kaw (1966). These expressions for the various components of the current density are substituted in the general wave equation and the solutions of the resulting differential equations along with the proper boundary conditions have been used to derive expressions for the amplitude of the electric vectors of these harmonic components in the reflected and transmitted waves from a plasma slab, surrounded by free space on both the sides.

Some numerical calculations have been carried out to investigate the variation of the amplitude of the electric vectors of these higher frequency components, in the reflected and transmitted waves, with the electron density and the thickness of the plasma slab; these results have been presented in the form of graphs. It is concluded that the intensity of harmonics is higher in the transmitted component than in the reflected component from the plasma slab and that, for a given magnitude of the electron collision frequency, there is an optimum value of the electron density for which the efficiency of harmonic generation is maximum. It is found, from the graphs, that the efficiency of second harmonic conversion is maximum when $0.80 < (\omega_p/\omega)^2 < 0.90$ (ω_p is the plasma frequency and ω is the frequency of the fundamental wave) and dimensionless plasma thickness ξ_0 ($= \omega z_0/C$, C is the velocity of light in vacuum) lies in the range $3.20 < \xi_0 < 3.50$ and that the efficiency of the third harmonic generation is maximum when $0.80 < (\omega_p/\omega)^2 < 1.00$ and $2.50 < \xi_0 < 3.40$. The variation of the efficiencies in these limits is not significant.

CHAPTER II OPTIMUM SECOND HARMONIC GENERATION IN INHOMOGENEOUS PLASMAS

In this chapter, using the expression for the second harmonic component of the current density in a plasma (due to an alternating electric field and gradients of electron density and temperature) derived by earlier workers, the author has obtained an expression for the second harmonic component in the reflected and transmitted waves from an inhomogeneous plasma slab when an electromagnetic wave is incident normally on it. This expression for the second harmonic component in the reflected and transmitted waves, has been used to calculate the optimum conditions under which the efficiency of second harmonic conversion in the reflected and transmitted waves from an inhomogeneous plasma slab, will be maximum. It is found that for a given value of electron collision frequency, there is an optimum value of the slab thickness and electron density for which the magnitude of the second harmonic component in the reflected and transmitted waves is maximum. The optimum magnitude of the second harmonic component in the transmitted wave is always greater than the optimum magnitude of the second harmonic component in the reflected wave. The maximum efficiency of second harmonic conversion in the transmitted wave occurs when $.70 < (\omega_p/\omega)^2 < .90$ for $3.2 < \xi_0 < 5.2$ while the maximum efficiency for the second harmonic conversion in the reflected wave occurs when $1.5 < (\omega_p/\omega)^2 < 3.0$ for $3.0 < \xi_0 < 4.0$. The variation of the second harmonic component with the electron collision frequency has also been studied and it is found that the optimum magnitude of the second harmonic component in both the reflected and transmitted waves increases with decreasing collision frequency and is maximum for electron collision frequency equal to zero.

CHAPTER III OPTIMUM THIRD HARMONIC GENERATION IN MAGNETOPLASMAS

In this chapter, the author has investigated the optimum third harmonic generation in an anisotropic homogeneous plasma; the anisotropy in the medium is considered due to the presence of an external magnetic field. Incorporating the proper form of the asymmetrical part of the electron velocity distribution function $\underset{\approx}{f_2}$ which is a tensor of second order, an explicit expression for the third harmonic component of the current density in a magnetoplasma has been derived. This expression for the current density component along with the solutions of the wave equation and appropriate boundary conditions are used to obtain an expression for the third harmonic component in the reflected and the transmitted waves from a plasma slab of finite thickness.

Some numerical calculations have been carried out to obtain the optimum conditions under which the efficiency of third harmonic conversion in a magnetoplasma is maximum; these results have been presented in the form of graphs. It is concluded that the magnitude of third harmonic component in the reflected and transmitted waves shows a resonance when the wave frequency of the fundamental wave becomes equal to the electron gyrofrequency. The optimum magnitudes of the slab thickness and the electron density for which the magnitude of the third harmonic component in the reflected and the transmitted waves is maximum, are given by ξ_c = .12 and $(\omega_p/\omega)^2$ = 4.0 respectively.

CHAPTER IV   OPTIMUM HARMONIC GENERATION IN SIMPLE MODEL SEMICONDUCTORS

In this chapter, the author has investigated the optimum thickness of a nondegenerate simple model semiconducting slab for which the magnitude of the second and the third harmonic components in the reflected and transmitted waves are maximum, when the semiconducting slab is subjected to the field of an electromagnetic wave and an external d.c. electric field. Using the Boltzmann transfer equation for the motion of free carriers in a semiconductor, the author has obtained expressions for the second and the third harmonic components of the current density in a simple model semiconductor at low temperatures; these expressions include some of the terms which have not been incorporated in the analysis of Sodha and Srivastava (1967) and Sodha et al. (1968). Substituting these expressions for the various components of the current density in the general wave equation and solving the resulting differential equations, the author has also derived the expressions for the electric vectors of these harmonic components in the reflected and transmitted waves from a semiconducting slab.

Some numerical calculations have been carried out to investigate the nature of variation of the electric vectors of the second and the third harmonic components, in the reflected and the transmitted waves from a semiconducting slab of germanium at 77°K, with the impurity concentration and the thickness of the slab; these results are presented in the form of graphs. It is found that appreciable second harmonics may be generated in the reflected and transmitted waves when $1.0 < \xi_{\omega} < 1.6$ and

$4.0 < (\omega_p/\omega)^2 < 6.0$ while the efficiency of third harmonic conversion is maximum when $\xi_0 = .80$ and $14.0 < (\omega_p/\omega)^2 < 16.0$ for the transmitted component and when $\xi_0 = .82$ and $12.0 < (\omega_p/\omega)^2 < 14.0$ for the reflected component.

CHAPTER V OPTIMUM SECOND HARMONIC GENERATION IN INHOMOGENEOUS SEMICONDUCTORS

Sodha and Gupta (1969) have studied the generation of second harmonic waves in an inhomogeneous semiconductor at low temperatures in the presence of electron density and temperature gradients. In their analysis, they considered only the electron velocity distribution function to be space dependent and neglected the space dependence of carrier relaxation time which at low temperatures ($\simeq 77^oK$) depends on the ionized impurity concentration and lattice temperature. In this chapter, the author has obtained an expression for the electric vector of second harmonic component in the reflected and transmitted waves from an inhomogeneous semiconducting slab at low temperatures considering carrier relaxation time to be space dependent. Optimum conditions under which the second harmonic component in the reflected and transmitted waves will be maximum, have also been obtained. It is found that for small concentrations of ionized impurity $[(\omega_p/\omega)^2 \simeq 2.0]$ the optimum slab thickness is large $[\xi_0 = 7.09]$ while for higher concentrations $[(\omega_p/\omega)^2 \simeq 13.0]$ the optimum slab thickness is found to be quite small $[\xi_0 = .797]$.

CHAPTER VI OPTIMUM SECOND HARMONIC GENERATION IN INDIUM ANTIMONIDE

In this chapter, the author has investigated the generation of second harmonic waves in a degenerate, low band gap and nonparabolic semiconductor at low temperatures when the semiconducting sample is subjected to an external d.c. electric field and an alternating electric field. Using Boltzmann transfer equation in k-space for the motion of free carriers in an indium antimonide sample, the author has obtained an expression for the second harmonic component of the current density taking into account the ionized impurity scattering of the free carriers. The expression for the alternating component of the current density, obtained above, is substituted in the general wave equation and the solutions of the resulting differential equations are used to obtain expressions for the electric vector of the second harmonic component in the reflected and transmitted waves from an InSb slab.

Some numerical calculations have been carried out, to investigate the optimum magnitudes of the slab thickness, for which the magnitude of the electric vector of the second harmonic component, in the reflected and transmitted waves, is maximum. It is found that the optimum magnitude of the second harmonic component in the transmitted wave is always greater than that in the reflected wave and that the efficiency of second harmonic generation is maximum when $0.75 < \xi_c < 0.95$ for the transmitted wave and when $0.78 < \xi_0 < 1.00$ for the reflected wave.

CHAPTER VII OPTIMUM SECOND HARMONIC GENERATION IN INHOMOGENEOUS INDIUM ANTIMONIDE

Solving the Boltzmann transfer equation in k-space for electrons, the author has obtained an expression for the second harmonic component of the current density in an inhomogeneous indium antimonide sample, subjected to an alternating electric field. This expression for the current density is used to solve the wave equation and hence to investigate the optimum thickness of the InSb sample, for which the conversion efficiency of the second harmonic component, in the reflected and transmitted waves, is maximum. It is found that the amplitude of the second harmonic component in the transmitted wave is always greater than that in the reflected wave and that the efficiency of second harmonic generation is maximum when $.76 < \xi_0 < .94$ for transmitted wave and when $.80 < \xi_0 < .98$ for the reflected wave.

Part II It is well known that when an amplitude modulated electromagnetic wave traverses an absorbing region of the ionosphere its index of modulation gets changed. The phenomenon of self-distortion or self-modulation of the wave in the presence of an external magnetic field is much more pronounced when the wave frequency becomes equal to the cyclotron frequency of the electron; this has been experimentally confirmed by

Cutolo (1952, 1953), Mitra (1954) and Bachynski and Gibbs (1969). Some attempts have been made to explain this phenomenon theoretically on the basis of self-interaction by Hibberd (1957), King (1959) and Ram and Kaw (1967) but none of the presently existing theories can explain the experimental observations of Bachynski and Gibbs (1969) according to which, the modulation index of the extraordinary mode of the amplitude modulated wave increases during its propagation through the medium and shows a resonance at gyrofrequency. In Chapters VIII and IX of the thesis, the phenomenon of self-modulation of ordinary and extraordinary mode of an amplitude modulated wave in a magnetoplasma has been studied using both the kinetic as well as the phenomenological approaches. In the kinetic approach the Boltzmann transfer equation for electrons for slightly ionized plasma has been solved for various components of the distribution function of electron velocities in the presence of the electric vector of carrier and two side bands of the amplitude modulated wave and the external magnetic field. These components of the distribution function are used to obtain the expressions for the nonlinear components of the current density for both the extraordinary and ordinary modes of propagation. This treatment is valid only when the modulation frequencies are much less than the carrier frequency. In the phenomenological approach the nonlinear current density for both the modes of propagation is obtained by solving the momentum transfer equation and the energy balance equation simultaneously taking time dependence of the electron temperature and hence, of the collision frequency into account. This treatment does not impose any restriction on the modulation frequency. Substituting the nonlinear current density thus obtained (by both kinetic and phenomenological approaches) in the wave equation, the resulting nonlinear second order differential equations are solved by the method of successive approximations. The solutions of these equations using the appropriate boundary conditions have been used to derive the expressions for the electric vector of the carrier and the two side bands of the wave propagating in the magnetoplasma. These expressions are then used to obtain the new index of modulation of the amplitude modulated wave for both the modes of propagation separately.

The titles and the summaries of the last two chapters of the thesis are given below:

CHAPTER VIII SELF-MODULATION OF AN AMPLITUDE MODULATED WAVE IN MAGNETOPLASMAS: ELEMENTARY THEORY

A phenomenological theory has been developed for nonlinear self-modulation of an amplitude modulated electromagnetic wave, propagating in a magnetoplasma along the direction of the external d.c. magnetic field, taking both the self-interaction and mutual interaction of the two modes into account; unlike presently available theories this theory is not restricted to small values of modulation frequency. It is shown that the ordinary mode of propagation becomes demodulated and extraordinary mode of propagation becomes over modulated during its propagation through the medium. Further, the change in modulation depth with the frequency of the wave is gradual for the ordinary mode and resonant for the extraordinary mode (around the gyrofrequency).

CHAPTER IX SELF-MODULATION OF AN AMPLITUDE MODULATED WAVE IN MAGNETOPLASMAS: KINETIC THEORY

In this chapter, using a rigorous theory, the author has investigated the phenomenon of self-modulation of an amplitude modulated wave propagating in a magnetoplasma along the direction of the magnetic field; the effect of self interaction and mutual interaction of both the ordinary mode and extraordinary mode on each other has been considered simultaneously. The variations of the new index of modulation with various relevant parameters has been studied and are in good agreement with the experimental observations of Bachynski and Gibbs (1969) and the theoretical predictions of the previous chapter.

The entire work has resulted in the following publications:

1. Optimum harmonic generation in plasmas. M.S.Sodha, B.K. Sawhney and R.L. Sawhney (1970) Brit. J. Appl. Phys. (In press).
2. Optimum second harmonic generation in inhomogeneous plasmas. M.S. Sodha and R.L. Sawhney (1970) Plasma Phys. (In press).
3. Self-modulation of an amplitude modulated wave in magnetoplasmas. M.S. Sodha and R.L. Sawhney (1970) Brit. J. Appl. Phys. (In press).

4. Self-modulation of an amplitude modulated wave in magnetoplasmas: Kinetic approach. M.S. Sodha and R.L. Sawhney (1970) Radio Science (Accepted for publication).

5. Optimum third harmonic generation in magnetoplasmas. M.S. Sodha, A. Singh and R.L. Sawhney (1970). Communicated.

6. Optimum harmonic generation in semiconductors. M.S. Sodha, B.K. Sawhney and R.L. Sawhney (1969). Communicated.

7. Optimum second harmonic generation in inhomogeneous semiconductors. M.S. Sodha and R.L. Sawhney (1970). Communicated.

8. Optimum second harmonic generation in indium antimonide. M.S. Sodha, B.K. Sawhney and R.L. Sawhney (1969). Communicated.

9. Optimum second harmonic generation in inhomogeneous indium antimonide. M.S. Sodha and R.L. Sawhney (1969). Communicated.

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