ELASTOPLASTIC-DAMAGE CONSTITUTIVE MODEL FOR CONCRETE

by

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① Elastoplastic damage
② Elastic model

CERTIFICATE

This is to certify that the thesis entitled **"Elasoplastic-Damage Constitutive Model for Concrete"** being submitted by **R. Raveendra Babu** to the **Indian Institute of Technology Delhi** for the award of degree of **Doctor of Philosophy** is a bonafide record of research work carried out by him under our supervision and guidance. The thesis work, in our opinion, has reached the requisite standard of fulfilling the requirement of the degree of **Doctor of Philosophy**.

The results contained in this thesis have not been submitted, in part or full, to any other University or Institute for the award of any other degree or diploma.

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Abstract

Concrete exhibits nonlinear inelastic behavior involving energy dissipation, irreversible deformations and stiffness degradation. The objective of the present dissertation is to propose a constitutive model for concrete capable of predicting the above mentioned aspects of its mechanical behavior. The approach followed here is essentially phenomenological aiming at describing the observed mechanical behavior of concrete undergoing small deformations.

In the proposed constitutive model, a new unified loading function has been proposed. The proposed loading function serves as a loading function both for plastic flow and stiffness degradation on damage. The loading function has been calibrated by using well established experimental data on failure and initial yielding of concrete. With loading beyond the initial loading surface, hardening occurs resulting in expansion of the loading surfaces from the initial yield surface to the failure surface. The evolution of loading surfaces with hardening involves their expansion and shape distortion without translation in the stress space. Due to stress variations within the initial yield surface, neither plastic flow nor any damage occurs and concrete exhibits isotropic linear elastic behavior. With further increase in stresses beyond initial yield, the value of the hardening function increases resulting in expansion of the loading surface as well as in the increase in the level of damage suffered by concrete.

A scalar damage parameter has been selected to measure the extent of damage suffered. In the present work, the isotropically damaged solid has been modeled as a bi-modular solid with different moduli of elasticity in tension and compression for open and closed cracks respectively. Depending upon the sense of the three principal strains, the damaged solid belongs any of the four possible cases of elastically distinct solids. When all the principal strains are of the same sense, the damaged material remains isotropic with elastic moduli decreasing with loading. The states of stress introducing principal strains of different senses render the material transversely isotropic. Under proportional variations of stresses within the

current loading surface, the material exhibits anisotropic linear elastic behavior whereas under arbitrary stress variations its behavior is anisotropic nonlinear elastic. Such nonlinearity is associated with rotation of principal stress/strain axes introduced by the non-proportional stress variations. However, for arbitrary variations of the normal stresses in the absence of shear stresses, the damaged concrete exhibits linear transversely isotropic elastic behavior. Using energy based approach, the model predicts continuous change in strains as well as strain energy with sudden change in the elastic constants when the stress variations result in change in sense of any of the principal strains. It has been found that only three elastic constants are required for this damaged solid.

During loading, the material exhibits strain increments depending upon applied stress increments, strain increments associated with incremental stiffness degradation suffered and plastic strain increments. Since for the damaged solid, the fourth rank tangent and secant compliance tensors are equal, the same compliance tensor as used for predicting total strains can be used for obtaining the strain increments introduced by applied stress increments. The incremental changes in these compliance tensor components with stress increments are used for determining the strain increments associated with incremental stiffness degradation. The plastic strain increments are obtained using associated flow rule.

The value of plastic modulus required for determining plastic flow depends on the choice of the hardening parameter. Here, both the effective plastic strain and plastic work have been chosen as hardening parameters. The hardening function-hardening parameter relation is considered to be valid for all load histories. Here, this unique relation has been derived from uniaxial compressive stress test. Realistic axial stress-axial strain and axial stress-lateral strain curves with peak at definite strains has been used in the present investigation. Also, the effect of stiffness degradation in uniaxial compression has been incorporated in the determination of the plastic modulus.

The proposed model has been used for predicting the material response to diverse stress histories including uniaxial compression and tension, equal and un-equal biaxial compression, triaxial compression, pure shear, etc. The behavior under unloading and reloading as well as stress reversal has also been investigated. The predicted material response at a point has been empirically validated. The model predicts stress-induced anisotropy, normal stress effects and nonlinear elastic behavior with non-proportional stress variations. Both of these effects arise out of stiffness recovery upon crack closure. Vertex effect has also been investigated. The theory of elastoplasticity without stiffness degradation as well as the theory of fracturing elastic solids without plastic flow is obtained as a special case of the proposed coupled damage elastoplasticity theory.

The proposed model is applicable only for concrete undergoing small deformations and is capable of predicting the pre-peak material behavior. Associative flow rule and scalar damage parameter have been chosen for analytical simplicity and computational ease. Theoretical significance of the approach followed and the practical relevance of the results obtained have been discussed.

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