

COHERENCE PROPERTIES OF PARTIALLY
POLARIZED RADIATION

by

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SUMMARY

The classical description of partially polarized light is studied in detail by Wolf¹, Parent and Roman² as well as Marathy³. They have used coherency matrix representation for partially polarized radiation. The coherency matrix J for the radiation field is given by

$$J = \begin{bmatrix} \langle E_x^* E_x \rangle & \langle E_x^* E_y \rangle \\ \langle E_y^* E_x \rangle & \langle E_y^* E_y \rangle \end{bmatrix} \quad (1)$$

where E_x and E_y are the components of the electric field in a direction mutually orthogonal to each other and at right angles to the direction of light wave. The trace of the coherency matrix represents the intensity of radiation

In the first chapter we review some of the basic concepts relating to the coherency matrix and the state of polarization. We obtain an expression for the intensity $I(\delta, \phi)$ when the light is passed through a compensator followed by a polarizer. If the intensity $I(\delta, \phi)$ is independent of δ and ϕ , we say that light is unpolarized. If $I(\delta, \phi)$ can be brought to zero for certain values of δ and ϕ , we say that light is completely polarized. For a partially polarized light the intensity $I(\delta, \phi)$ varies between $I_{\max}$ and $I_{\min}$ for different values of δ and ϕ . The degree of polarization may be defined as

(2)

$$P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}},$$

$$= \sqrt{1 - \frac{4|J|}{(\text{Tr } J)^2}}. \quad (2)$$

It is seen that for a completely polarized radiation the matrix element J_{ij} must factorize in the form $\alpha_i \alpha_j^*$.

In the second chapter we consider the transformation properties of density operator when the light beam is passed through a compensator, rotator or a polarizer. In particular, we obtain expressions for the transformed density operator and its diagonal coherent state representation in the above mentioned cases. Defining the unpolarized light as one for which the statistical properties (and hence the density operator) remain unchanged under rotation and arbitrary phase changes between the two orthogonal plane polarized components, we obtain density operator and its diagonal coherent state representation for unpolarized radiation^{4,5}. Analogously a completely polarized radiation is defined as one for which one of the polarization modes can be brought to ground state by introducing a suitable phase change between two orthogonal polarization components followed by a rotator⁶. Using this property we obtain the general expression of the density operator for a completely polarized radiation. We also consider, how the characteristic function of light beam changes when it is passed through a

compensator followed by a rotator and obtain the characteristic functions of unpolarized and completely polarized radiations. As an illustration we consider the case of thermal radiation. We show in general that the coherent state representation of an arbitrary thermal radiation can uniquely be expressed as a convolution of coherent state representations of unpolarized and completely polarized thermal radiations. The degree of polarization obtained by comparing the intensity of polarized portion of this decomposition with the total intensity agrees with Eq. (2) given above. It is worth noting that such a decomposition is not possible in the general case. Next, we consider the intensity transmittance of the system when the light is passed through various physical devices e.g. rotator, compensator, polarizer or any combination of these. The variation of photoelectron counting distribution is also studied in these cases.

In chapter three we consider the density operator for the radiation field reflected and transmitted at a boundary between two homogeneous media of different optical properties. We also consider the case when the direction of propagation of the reflected and transmitted waves is at right angle to each other (Brewster angle incidence). In this case we find that the density operator for the reflected radiation corresponds to that of linearly polarized

light as is expected. Again the case of thermal light is considered as an illustration.

In chapter four we consider the phase problem of optical coherence theory for a linearly polarized radiation. According to Wiener-Khintchine theorem, the complex degree of temporal coherence $\gamma(\tau)$ and the spectral density $g(\nu)$ for a light beam form a fourier transform pair⁷. Hence a measurement of the temporal coherence function will give information about the spectral profile of the light beam. However, only $|\gamma(\tau)|$ is measured in practice. The phase of $\gamma(\tau)$ can in principle be measured by locating the position of interference fringes. Unfortunately such measurements are extremely difficult to perform. Using the analytic properties of $\gamma(\tau)$, the phase of $\gamma(\tau)$ can also be measured by putting an exponential filter at the source and performing visibility measurements⁸. However it is extremely difficult to construct an exponential filter in the optical frequency range. In this chapter we propose a method for the construction of exponential filter by using the evanescent wave coupling between two prisms.

The last chapter of this thesis deals with the determination of spectral profile of partially polarized radiation⁹. In determining the spectral profile of linearly polarized radiation, the electric field is assumed to be a scalar analytic signal. If we consider the vector proper-

(5)

ties of radiation field, the temporal coherence for the partially polarized radiation is characterized by the coherence matrix $\gamma(\tau)$,

$$\gamma(\tau) = \begin{bmatrix} \gamma_{xx}(\tau) & \gamma_{xy}(\tau) \\ \gamma_{yx}(\tau) & \gamma_{yy}(\tau) \end{bmatrix} \quad (3)$$

It is shown that, if in this case we know the phase of ^{one of} the components of coherence matrix say $\gamma_{xx}(\tau)$, the complete coherency matrix can be obtained by fringe visibility measurements using a suitably oriented polarizer followed by a compensator. Hence the spectral density matrix which is the Fourier transform of $\gamma(\tau)$ can be obtained.

In appendix A, we use the angular momentum states to characterize the unpolarized and completely polarized radiations. In appendix B, we calculate the characteristic function for the unpolarized and completely polarized radiations. In appendix C, we show that the characteristic function of an arbitrary thermal radiation can be uniquely written as the product of characteristic functions of unpolarized and completely polarized radiations. Equivalently, we can write the coherent state representation of an arbitrary thermal radiation as a convolution of coherent state representations of unpolarized and completely polarized radiation.

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