GRAPH THEORETIC APPROACH TO TOPOLOGY CONTROL IN WIRELESS SENSOR NETWORKS

by

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Certificate

This is to certify that the thesis entitled **"Graph Theoretic Approach to Topology Control in Wireless Sensor Networks"** submitted by **"Mr. Pushparaj Shetty D."** to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of the original bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

A wireless sensor network (WSN) consists of a collection of battery powered sensors. Each sensor is integrated in a single package with low power signal processing, computation, and a wireless transceiver. Since the battery of each sensor is of limited capacity and it is not possible to replace the battery always, energy conservation is a critical issue in order to increase the lifetime of a sensor network. A WSN is modelled by a complete weighted directed graph, $G = (V, E)$, with each sensor representing a vertex. A directed edge from vertex u to v exists in G if the sensor node v is in the transmission range of u. A cost function $c : V \times V \rightarrow \mathbb{R}^+ \cup \{\infty\}$ is associated with each arc, such that $c(u, v)$ is the power emission necessary for node u to send packet to node v. If $c(u, v) = \infty$, then node u cannot communicate to node v directly. If both edges (u, v) and (v, u) are present in G, then these two edges are replaced by an undirected edge uv in G. The power of a node v in G is the maximum cost of its incident edges, that is $P_T(v) = \max\{w(uv)|uv \in E(G)\}$ and the sum of powers of all nodes $v \in V$ is the total power of the graph, that is $P(T) = \sum_{v \in V} P_T(v)$. Given a network communication graph, the problem of computing a subgraph with specific desired properties such as connectivity, short stretches, sparsity, low interference, low degree node etc. is called *topology control* problem in WSN.

In this thesis we study the *strong minimum energy topology* (SMET) problem.

The SMET problem is to find a spanning tree T of G such that $P(T)$ is minimum. The SMET problem is known to be NP-complete. We study several special cases of the problem and prove the NP-completeness of these special cases. We propose heuristics for SMET problem and compare the results of proposed heuristics with existing heuristics.

In the context of hierarchical sensor networks, we define *k-Strong Minimum Energy Hierarchical Topology* (k-SMEHT) and *Strong Minimum Energy 2-hop Rooted Topology* (2h-SMERT) problems. We prove that k-SMEHT Problem is NP-hard for arbitrary k and propose a $\frac{k+1}{2}$-approximation algorithm for k-SMEHT problem for a fixed constant k. We prove that the 2h-SMERT is NP-hard and also APX-hard. We then show that 2h-SMERT is not approximable within a factor of $\frac{1}{2}(1-\epsilon)\ln n$ unless NP $\subseteq DTIME(n^{\log\log n})$.

An important issue in wireless communication is interference, where communication between two parties is affected by transmissions from a third party. We propose two new models for measuring the interference and present algorithms for *minimum interference strong bidirectional topology* (MISBT) under these models. We prove that our algorithms under these model gives optimal result for Min-max interference. The average interference obtained by our algorithms under these models are at most twice the optimal. We propose a local search based heuristic for minimizing both power and interference.

Fault tolerance is an important property of a network, which demands two or higher connectivity. The *minimum power two-connected subgraph* (MP2CS) problems seeks to find a subgraph H of G such that H is 2-connected and the power $P(H)$ is minimum. The MP2CS problem is proved to be NP-hard. We provide an alternate NP-hard proof for MP2CS problem. We propose a heuristics for MP2CS problem and a special case of MP2CS problem called the minimum power k-backbone 2-connected subgraph (MPkB2CS) problem.

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