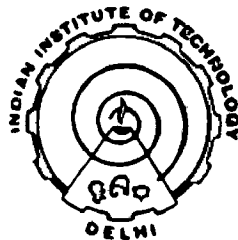


**SOME MULTIGRID SOLUTIONS FOR BOUNDARY  
VALUE PROBLEMS AND HIGHER ORDER ADI  
METHODS FOR INITIAL VALUE PROBLEMS  
IN SPHERICAL COORDINATES**

**By  
ANU GOYAL  
Department of Mathematics**

**Thesis Submitted  
in fulfilment of the requirements  
of the degree of  
DOCTOR OF PHILOSOPHY**



**to the  
INDIAN INSTITUTE OF TECHNOLOGY, DELHI  
HAUZ KHAS, NEW DELHI-110016, INDIA.  
1992**

TO MY  
PARENTS.



## CERTIFICATE

This is to certify that the thesis entitled " Some Multigrid solutions for Boundary Value Problems and Higher order ADI methods for Initial Value Problems in Spherical Coordinates ", which is being submitted for the award of the degree of Doctor of Philosophy (Mathematics), to the Indian Institute of Technology, New Delhi, is a bonafide record of research work done under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other university or Institute for the award of any degree or diploma.

*S. R. K. Iyengar*  
(S.R.K. Iyengar)

## ACKNOWLEDEMENTS

I wish to express my profound sense of gratitude to Prof.S.R.K. Iyengar, for his excellent guidance, and for being a constant source of inspiration , support and encouragement. No words can express what I have gained under his perfect and untiring supervision.

I would also like to express my sincere gratitude to Prof. M.K. Jain , Prof. M.M. Chawla, and Prof. R.K. Jain for their teaching and help rendered to me. Their suggestions and support have been invaluable.

My thanks are also due to Prof. P.K. Bhattacharya, Prof. K.N. Mehta, and Dr. Raj Ahuja for the courses taught by them at the M.Sc and Pre Ph.D levels, which have proved to be helpful in my research work. I also thank the authorities of I.I.T. Delhi, the Head of the Department of Mathematics, and the Head of the Computer Service Centre for the research facilities provided to me.

I am thankful to my friends Neelima, Dr.Pragya, Shobha, Viveka, Sangita, Meera, Vatsla, Anju and Neela for their help and cooperation in the preparation of this thesis.

Lastly to my parents and brother Deepak, I owe more than I can possibly express for their love, encouragement and sacrifices throughout my research period.

I. I. T DELHI

Anu Goyal  
ANU GOYAL  
16-1-1992

## CONTENTS

CHAPTER 1	SURVEY OF RELEVANT LITERATURE	
1.1	Introduction.	1
1.2	Multigrid and some related terminology.	3
1.3	Multigrid Methods.	11
1.4	Some Grid transfer Patterns.	15
1.5	Restriction and Prolongation Operators.	19
1.6	Smoothing Procedures.	23
1.7	Defect Correction in Multigrid solutions.	25
1.8	Application of Multigrid Methods for Non-Linear Boundary Value Problems.	28
1.9	Brief History of Multigrid Methods.	30
1.10	Higher Order Finite Difference Methods.	32
1.11	Methods for the Poisson Equation in Polar Cylindrical Coordinates.	41
1.12	Review of the Contents of the Present Thesis.	44
CHAPTER 2	COMPARISON OF S AND V-CYCLES IN MULTIGRID FOR THE LINEAR ELLIPTIC EQUATION IN VARIABLE COEFFICIENTS.	
2.1	Introduction.	53

2.2	Smoothing Factors for the Relaxation Procedures for the Poisson Equation: .	55
2.3	Multigrid Methods for the Poisson Equation.	59
2.4	Defect Correction for Poisson Equation.	64
2.5	Multigrid Methods for the General Linear Elliptic Equation.	66
2.6	Conclusions.	73

CHAPTER 3 MULTIGRID IMPLEMENTATION FOR THE THREE  
DIMENSIONAL POISSON EQUATION IN POLAR CYLINDRICAL  
COORDINATES.

3.1	Introduction.	75
3.2	Multigrid Method.	77
3.3	Numerical Experiments.	85
3.4	Conclusions.	89

CHAPTER 4 HIGH ORDER ADI METHODS FOR HEAT TRANSFER PROBLEMS  
IN SPHERICAL COORDINATES.

4.1	Introduction.	90
4.2	Governing Equation and Limiting Cases.	92
4.3	Higher Order Methods.	94
4.4	ADI Methods for Limiting Cases.	104
4.5	Stability Analysis.	108
	Appendix.	118

CHAPTER 5 A HIGH ORDER ADI SCHEME FOR TRANSIENT HEAT  
TRANSFER TO A DROPLET SUSPENDED IN AN ELECTRIC FIELD.

5.1	Introduction.	122
5.2	Formulation.	124
5.3	Higher Order Approximations.	129
5.4	Quadrature Formula for Eq.(5.2.8).	136
5.5	Solution Procedure.	137
5.6	Computational Results.	140

CHAPTER 6 HIGH ORDER DIFFERENCE SOLUTION FOR THE CONJUGATE  
UNSTEADY HEAT TRANSFER FROM A SPHERICAL DROPLET AT LOW  
PECLET NUMBERS.

6.1	Introduction.	147
6.2	Problem Details.	150
6.3	Implementation of the ADI Methods.	152
6.4	Results and Conclusions.	163

REFERENCES	171
------------	-----