ON MODELING AND PARAMETER ESTIMATION OF NONGAUSSIAN AND NONMINIMUM PHASE ARMA PROCESSES

by
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CERTIFICATE

This is to certify that the Ph.D dissertation entitled **'On Modeling and Parameter Estimation of NonGaussian and Nonminimum phase ARMA Processes'**, submitted by KATAM CHANDRASEKHAR, is a bonafide record of the research work done by him, during the period from July 1993 to August 1998, under my supervision. The results obtained in this dissertation have not been submitted for the award of any degree or diploma so far.

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ABSTRACT

In order to understand, analyze and sometimes reproduce different natural and man made phenomena around us, we make observations of one kind or the other sequentially over time. All these phenomena are generally called as time series. One of the fundamental issues of concern in time series analysis is that of modeling of a given random sequence, to extract the useful information hidden in these observations. The Gaussian ARMA models have been used widely so far for this purpose; due to their simplicity, ease of implementation and analytical tractability, even for the applications concerning nonGaussian (possibly nonminimum phase) processes . Recently, nonGaussian signal processing methods, which make use of Higher Order Statistics (HOS) have attracted widespread attention due to their ability to reveal the phase of nonminimum phase processes. The technological progress in the field of computers, which has facilitated the implementation of computationally intensive models, has provided further impetus to these developments. And also higher order statistics and spectra of a stochastic process provide additional information regarding its character and parameters characterising it, to what can be obtained only from the second order statistics and power spectrum (as is the case in Gaussian models).

The problem addressed in this thesis is the filtering and parameter estimation of nonGaussian and nonminimum phase ARMA processes. Since the optimum prediction of the future in the mean square sense for these processes is a nonlinear function of the past, we develop a framework in terms of the predictor algebra associated with the entire past (i.e. all finite $L_2(\mu)$, B_t measurable maps, where B_t is the σ-algebra generated by entire past at time 't', and μ being the probability measure) to tackle the nonlinear prediction problem. Our approach to solve a linear problem by going into the nonlinear domain may sound unnatural and may look like a backward step apparently. But by the very nature of the nonGaussian and more importantly nonminimum phase the process; the optimum predictor is a nonlinear

function of the past data; and hence our approach seems to provide a more natural framework for this situation; as has been vindicated by the results obtained in the thesis. Also the problem that is addressed here, i.e. the estimation of ARMA parameters, is an inherently nonlinear one. If the input noise is given, only then the problem is linear. So the ARMA parameter estimation with unknown white Gaussian noise (WGN) input itself is nonlinear. We, in our work, have suggested alternate parameterisation where the estimation of AR and MA parameters gets decoupled and estimation becomes possible in a linear framework.

Since more statistical information is being used in parameter estimation using HOS, one can expect to obtain better results in terms of accuracy, i.e., estimates with lesser error variance as compared to the second order case (though at the cost of increased computational complexity). To extract the information hidden in the nonlinear functions of the samples of the process (through which we extract the higher order statistical information of the process); and still operate in a linear framework), we propose to expand the observation space concerned to include the nonlinear terms also. This is useful in not only extracting phase information, but also takes us closer to the optimum estimator in the nonGaussian case.

We derive here a new representation for nonGaussian and nonminimum phase ARMA processes, termed here as, Generalised Predictor Space Representation (GPSR); using the algebraic structure of the $L_2(\mu)$ space associated with the process. Let B_t^- be σ-algebra generated by all the random variables associated with a stationary stochastic process { y(t) } which are past at time t, i.e. { $y(t+T) \mid T < 0$ }. Further let B_t^+ denote the σ-algebra generated by all the future and present, i.e. $\{y(t+T) \mid T \geq 0\}$. The appropriate algebras are defined as follows [57],[46]:

H_t^- = Algebra of all B_t^- measurable Borel maps with finite $L_2(\mu)$ norm.

= Algebra generated by all the infinite past of y(t) at time t.

H_t^+ = Algebra of all B_t^+ measurable Borel maps with finite $L_2(\mu)$ norm.

= Algebra generated by the present and future samples of y(t).

$P_t = < H_t^+ \mid H_t^- >$ = The algebra generated by all the predictors called here the Predictor Algebra.

We define innovation at time 't' as

$$z(t) \triangleq y(t) - E\{ y(t) / B_t^- \} = y(t) - y(t) \mid H_t^-$$

{ where the projection means conditional expectation } (6)

First by defining innovation as above, and making use of Predictor Algebra and its properties, we obtained a Generalised Predictor Space Representation for the nonGaussian and nonminimum phase ARMA processes. By approximating the underlying space in the above representation depending on upto what order statistics is being used, one can easily see that there exists a natural nesting of subspaces as the order of nonlinearity increases. Here we present some numerical results for the case of nonGaussian and nonminimum phase ARMA processes, by using canonical correlation method, where we make use of the second order nonlinear terms of the process. The parameter estimates obtained in this case amply demonstrate our claim of the utility of the proposed theory to nonGaussian and nonminimum phase ARMA processes. These results also indicate the type of performance expected from the RLS algorithm developed later for the same case.

To get further insight into GPSR for ARMA processes, we derive the relevant results for the Generalised Predictor Space of nonlinearity order two and memory N, where N corresponds to the order of the nonGaussian, nonminimum phase ARMA(N,N) process. This is a special case of GPSR when the underlying space is restricted to contain linear and second order nonlinear terms only. Here we have assumed that the process has finite fourth order cumulants, justifying the selection of nonlinearity order two. Another criterion for selecting the nonlinearity order is the ease of presenting the results without loss of generality. We have also shown the finite dimensionality of the predictor space given the memory and order of the nonlinearity, when the underlying process is ARMA.

Next we take up the development of time recursive ladder type algorithm for nonGaussian ARMA filtering, by utilising the Generalised Predictor Space Representation of nonlinearity order two and zero memory. Here we derive a Projection Operator Update formula and the associated inner product update formula, which is particularly suited for this case. By making use of the Generalised Predictor Space Representation (GPSR) (of nonlinearity order two and zero memory) of the process in conjunction with the inner product update relation, we have developed a fast recursive least squares algorithm for the filtering of the process (i.e whitening filter) and also estimating the AR parameters of the process from the resulting structure. As the predictors and the innovations (of both first and second order) appearing in the equivalent representation of the ARMA process are orthogonal to each other, AR parameter estimation can be decoupled from the MA parameter estimation. The recursive least squares algorithm developed for filtering nonGaussian and nonminimum phase ARMA process, is shown to result in a nonlinear lattice like filter structure.

Next we take up the issue of estimating the AR parameters in the least squares sense from the nonlinear whitening lattice filter. For this purpose, we derive some pseudo inverse update formulas and use them for the development of the algorithm for AR parameter estimation from the filter coefficients. We have used two existing methods for MA parameter estimation, from the resulting AR compensated sequence (i.e. innovation sequence) from the lattice filter. The numerical results are presented which substantiate the theory.

Finally we address the extension of this results to nonlinear system identification. Also we propose a scheme for ARMAX modeling, which results directly from the recursive least squares algorithm developed for the nonGaussian and nonminimum phase ARMA processes.

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(Katam Chandrasekhar)

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