

**IMPROVED ZERO ORDER TECHNIQUES
FOR
STRUCTURAL SHAPE OPTIMIZATION
USING F.E.M.**

**BY
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CERTIFICATE

This is to certify that the thesis entitled 'Improved Zero Order Techniques for Structural Shape Optimization Using F.E.M.' being submitted by Mr. Ali Ghoddosian to the Indian Institute of Technology, Delhi for the award of the Degree of Doctor of Philosophy in Applied Mechanics is a record of bonafide research work carried by him under my supervision and guidance.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

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(A. Ghoddosian)

ABSTRACT

Most of research in structural optimization is focused on interfacing analysis programs directly with first order optimization algorithms. The algorithms require gradients of constraints on structural responses such as stresses and displacements. Calculating sensitivity by finite difference is computationally expensive when the total number of design variables is large. On the other hand analytical methods for sensitivity calculations are computationally more efficient, but considerable coding is required by experts in the field to implement them. These difficulties in obtaining first order information motivated the current work of a study on zero order approaches in order to modify and improve some of these techniques by imposing the latest developments in this field and proposing new techniques to tackle the difficulties of sensitivity analysis. And on the other hand enhancing the efficiency of original methods by strengthening their shortcomings. The thesis attempts to address the following objectives, which have not received serious attention.

1. Optimum design of two-dimensional elastic bodies subjected to static loads based on proposed two-phase, shape and thickness, optimization.

In two-dimensional structural analysis, no attention is paid to thickness direction because of theoretical and practical difficulties. From theoretical point of view, stresses or strains in thickness direction are not significant in comparison with the ones in other directions and from practical point of view, manufacturing of structure with optimum shape is very difficult. But in structural optimization when the objective is to minimize the weight of the structure, thickness becomes important. On the other hand due to the development of manufacturing process and equipment, all those barriers in the way of

manufacturing the structure with optimum shape are removed. So by taking into account the above mentioned points, here we have proposed a two-phase optimization. In the first phase, shape of the body is optimized and in the second phase, optimization will be performed with respect to its thickness. It is shown that by applying this technique not only lighter structure will be obtained but also stress distribution will be more uniform. For the first phase a mathematical programming method based on state of the art for each module is used which comprises the B-spline curve for describing the boundaries, finite element method as analyser, either F. E. or F. D. methods is used for sensitivity calculations and method of moving asymptotes is employed for optimization module. For the second phase variable thickness within each element is considered and modified pattern transformation method (zero order method) with variable reference stress is applied for thickness optimization. Three standard examples, square plate with a hole, fillet and connecting rod are optimized by proposed method. It is demonstrated that the optimum structure by the proposed method is lighter with more uniform stress distribution than the optimum of any single-phase method.

2. New method for thickness optimization of 2D bodies with severe thickness changes.

When the optimum thickness distribution, of two-dimensional bodies have got sharp change in thicknesses, i.e. thickness distribution is not smooth, applying two-dimensional stress analysis will cause some errors. In this case for obtaining the accurate results in thickness optimization, use of three-dimensional shape optimization is inevitable. So, for this particular case (thickness optimization with severe thickness variation), we have proposed a pattern transformation method integrated with three-dimensional stress

analysis. The suggested method is much more efficient than conventional 3D optimization, with respect to computation – time and simplicity of coding.

3. Proposing improved pattern transformation method for thickness optimization of axisymmetric structures and shells.

In this part of the work we have proposed an improved pattern transformation method. In this method design element module is adopted from the mathematical programming techniques and modified stress ratio criterion by proper blending function from pattern transformation method is used for shape modification. Due to these modifications the following outstanding features of the new method can be highlighted 1) Tackling the main disadvantage of pattern transformation method, i.e. roughness of obtained optimum boundary shape. 2) Decreasing the number of design variables by assuming the master nodes as design variables. 3) In the absence of sensitivity and optimization modules from mathematical programming method and using improved pattern transformation method, the computation-time of the program is decreased significantly.

On the whole, by taking into account the efficiency of each method and limitation of applicability of any optimization technique to a specific class of problems, it is emphasized that by modifying pattern transformation method not only the applicable range of this method is increased but also the efficiency of the original method is improved. Two softwares based on this technique are developed for axisymmetric bodies and shells. The efficiency of proposed method for axisymmetric bodies is demonstrated by optimizing two axisymmetric pressure vessels under internal pressure. And two optimized shell examples, a cylindrical roof under uniform distributed load and

cylindrical tank subjected to hydrostatic load, show the versatility and efficiency of the suggested method in this field. In each case both inner and outer surfaces can be considered as design variables (moving surfaces), independently.

4. Proposing curvature method for shape optimization of shell structure.

Using zero order method for thickness optimization is an effective method, which is employed in our work, as it is mentioned earlier. But the use of this technique in shape optimization encounters with more difficulties. The most important one is the proper direction of movements for design variables are not known. Based on the work done recently for shape optimization of two-dimensional bodies by using the curvature function, we have proposed a new method, for reduction of peak stresses in shell structures. In this method shape design variables are considered. Since developed stresses in shells are mainly function of two parameters, thickness and curvature, by assuming a linear relation between the stress and curvature, approximate sensitivity can be calculated. This analysis defines the proper directions of the movements, for design variables. Improved pattern transformation method is used for modification of the shell's shape. Taking the advantage of design element technique not only paves the way for achieving a smooth optimum shape but also prevents the distortion of F. E. mesh. A software based on this theory is developed and two cylindrical shells under static loads are optimized. Efficiency and simplicity are the two fundamental features of the proposed method.

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