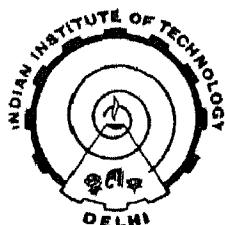


STUDIES IN PARAMETER ESTIMATION USING LAGUERRE - POLYNOMIAL APPROACH

A Thesis
Submitted for the degree of
DOCTOR OF PHILOSOPHY
in Electrical Engineering

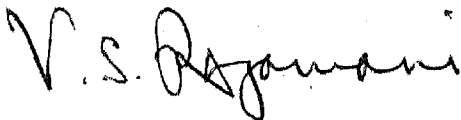
by
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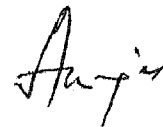
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October, 1985

CERTIFICATE

This is to certify that the thesis entitled, "Studies in Parameter Estimation using Laguerre-Polynomial Approach", being submitted by Mr. V. Ranganathan for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of the original bona fide research work carried out by him under our guidance and supervision. The results contained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.



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V. RANGANATHAN

ABSTRACT

The thesis deals with the problem of parameter estimation in lumped and distributed parameter systems, using Laguerre polynomials. The approach followed starts by eliminating all derivatives in the mathematical model of the system by successive integration, i.e. it converts the differential equation into integral one. This successive integration procedure introduces the unknown initial and boundary conditions explicitly in the mathematical model. These conditions are assumed in the form of truncated Laguerre series whose coefficients are to be determined. Clearly, once these coefficients are determined, the truncated Laguerre series will then yield an estimate of the initial and boundary conditions. This way, the present identification problem reduces to that of determining the coefficients of the differential equation and the coefficients of the aforementioned truncated Laguerre series. Subsequently, the input and output signals are expanded in truncated Laguerre series and are introduced into the integral equation where upon equating coefficients of like Laguerre polynomials, a linear system of equations in both the unknown model parameters and unknown Laguerre coefficients of the initial and boundary conditions is derived. We make use of the bidiagonal property of Laguerre polynomial operational matrix to make the above linear equations recursive and then solve them using Greville's algorithm to get an estimate of the parameters of the model. Our method is useful for estimating parameters even in an unobservable system.

because of the application of Greville's algorithm.

In all seven problems have been considered. In the first, a recursive least-square estimation algorithm has been developed for finding out the coefficients of the Laguerre polynomial expansion of a signal, when a truncated series of polynomials are used to represent the signal. We have shown how the same algorithm can be used for both the single variable and double variable functions. Numerical results are given to illustrate the algorithm.

The second problem considers parameter estimation of linear distributed system. The method is illustrated by a numerical example. The problem of parameter estimation in bilinear lumped systems and a class of non-linear lumped systems with fractional powers in non-linearity is considered next.

The fourth problem considers parameter estimation in non-linear distributed system. Here we have included a case study. Subsequently, use of Laguerre polynomials has been extended to cover the identification of parameters of a system described by a linear differential equation with coefficients of finite order polynomials in time t . Also we have used Laguerre polynomials to identify the parameters of a system described by a linear partial differential equation (distributed system) with coefficients of finite order polynomials in time t .

Lastly, a general algorithm is presented for identification of the parameters in the transfer function matrix of a multi-input-multi-output (MIMO) system, using Laguerre polynomials. An example is included to illustrate the applicability of the proposed method.

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