BOTTLENECK EXTREMA

THESIS SUBMITTED
IN FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

by

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to the

INDIAN INSTITUTE OF TECHNOLOGY, DELHI
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CERTIFICATE

This is to certify that the thesis entitled

BOTTLENECK EXTREMA

which is being submitted by **Kanchan Mathur** for the award of the degree of **DOCTOR OF PHILOSOPHY** in **MATHEMATICS** to the **INDIAN INSTITUTE OF TECHNOLOGY, DELHI**, is a bonafide record of research work done under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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To my parents,

with love and thanks

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ABSTRACT

This thesis, entitled "Bottleneck Extrema", is a study of bottleneck programming problems and their variants, including the bilevel bottleneck programming problem. The subject matter has been divided into four chapters.

Chapter 1 is an introductory chapter. The first two sections of this chapter contain brief surveys of the work carried out by various authors in the fields of bottleneck and bilevel programming respectively. The third section contains a brief summary of the work done by the author and presented in the remaining three chapters of the thesis.

Chapter 2 contains four sections. The first two sections present two methods of ranking of the basic feasible solutions of the bottleneck linear programming problem. The third section in this chapter presents an algorithm for solving the bottleneck programming problems, in which the objective function is the sum, ratio or product of two bottleneck functions. The fourth section provides an extension to this algorithm, to find the k^{th} $(k \geq 2)$ best solutions of the problems being studied.

Chapter 3 consists of two sections. In the first, the bicriteria bottleneck programming problem, in which both the objectives are bottleneck functions, is studied. In the next section, a combinatorial optimization problem in which the objective function is the sum of a linear and a bottleneck function, is studied.

Chapter 4 deals with bilevel programming problems with bottleneck policy objectives. The first section in this

chapter studies the salient features of the bilevel bottleneck programming problem, and compares them with those of the bilevel linear programming problem. The next section presents an algorithm to solve the bilevel bottleneck programming problem. The third section here introduces the bilevel fractional programming problem, in which the inner objective function is a linear fraction. The fourth section studies the cases when the outer objective function in the bilevel fractional programing problem is a linear function, a ratio of two linear functions, and a bottleneck function.

Numerical examples are given to illustrate the various algorithms developed in the thesis. The bibliography is provided at the end of the thesis and the numbers in the brackets [ ] refer to the papers/books mentioned there. The numbering of various results is such that the first number represents the chapter, the second number represents the section of the chapter and the third number is the serial number of the result in that section. Equations/inequations are represented by the numbers such that the first number represents the chapter and the second represents the serial number of the equation/inequation occurring in that chapter.

PUBLICATIONS

1. Bicriteria Bottleneck Linear Programming Problem, **Optimization**, 28, 1993, 165-170.
2. Combinatorial Min-Max-Min-Sum Optimization over a Polytope **Cahiers du Centre D'Etudes de Recherche Operationnelle**, 35 (3-4), 1993, 173-186.
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6. A Bilevel Bottleneck Linear Programming Problem, to appear in **European Journal of Operational Research**.
7. On Bilevel Fractional Programming, to appear in **Optimization**.
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CONTENTS