THE OPEN-POINT, BI-POINT-OPEN AND BI-COMPACT-OPEN TOPOLOGIES ON $C(X)$

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THE OPEN-POINT, BI-POINT-OPEN AND BI-COMPACT-OPEN TOPOLOGIES ON $C(X)$

by

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Dedicated to

My Family

Certificate

This is to certify that the thesis entitled **THE OPEN-POINT, BI-POINT-OPEN AND BI-COMPACT-OPEN TOPOLOGIES ON** $C(X)$ submitted by **Ms. Anubha Jindal** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by her under my guidance and supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

New Delhi
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Abstract

In this thesis, we introduce and study three new topologies, 'the open-point topology', 'the bi-point-open topology' and 'the bi-compact-open topology' on $C(X)$, the set of all continuous real-valued functions defined on a Tychonoff space X. The open-point, bi-point-open and bi-compact-open topologies on $C(X)$ are denoted respectively by h, ph and kh. The spaces $C(X)$ equipped with the open-point topology h, the bi-point-open topology ph and the bi-compact-open topology kh are denoted respectively by $C_h(X)$, $C_{ph}(X)$ and $C_{kh}(X)$. After giving definitions of the open-point, bi-point-open and bi-compact-open topologies on $C(X)$, we give some bases for these spaces. Then we study some special maps defined on these spaces and compare these topologies among themselves and with the well-known topologies on $C(X)$ such as the point-open topology p and the compact-open topology k. Then we characterize the following topological properties of $C_\tau(X)$, where $\tau = h$, ph, kh in terms of the topological properties of X:

(i) Separation Axioms.

(ii) Submetrizability.

(iii) Cardinal functions such as character, pseudocharacter, tightness, extent, weight, density and cellularity.

(*iv*) Various countability properties such as first countability, Lindelöf property, second countability and separability.

(*v*) Metrizability.

(*vi*) Completeness properties such as complete metrizability, Čech-completeness, pseudocompleteness and the property of being a Baire space.

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