

**A THESIS ON  
NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS**

**By  
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**Submitted to the Indian Institute of  
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of the Degree of Doctor of Philosophy  
in Mathematics  
1968**

## **C E R T I F I C A T E**

**This is to certify that the thesis entitled  
"Numerical Solution of Ordinary Differential Equations"  
being submitted by Mr. Rajinder Kumar Jain to the  
Indian Institute of Technology, Delhi for the award of  
the degree of Doctor of Philosophy in Mathematics,  
is a record of bonafide research work carried out by  
him. Mr. Rajinder Kumar Jain has worked under my  
guidance and supervision for the last two years. His  
thesis has reached the standard fulfilling the  
requirements of the regulations relating to the degree.**

**The results contained in this thesis have not  
been submitted in part or in full, to any other  
university or Institute for the award of any degree  
or diploma.**

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## ACKNOWLEDGEMENTS

I feel great pleasure in expressing my regards and gratitude to Prof. M.K. Jain, Ph.D., D.Sc., Head of Mathematics Department, Indian Institute of Technology, New Delhi, under whose supervision and guidance I had the privilege to work for the last two years. I am indebted to him for his valuable suggestions and comments, encouragement and inspiration, which I received from him from time to time.

I am obliged to the authorities C.S.I.R., for awarding the scholarship, and thus providing an opportunity to me for higher studies.

I am thankful to Prof. R.N. Dogra, Director, Indian Institute of Technology, New Delhi, for his help in providing all research facilities to me over here.

I wish to thank my colleagues here, who helped me in several ways. In particular I am thankful to Miss Swadesh Juneja and D. Subramaniam for their kind help and cooperation in computation done for this thesis.

My thanks are due to my father Shri B.R. Jain for his blessings, and my wife Mrs. Vinod Jain who stood by me in all strains and stresses.

Finally I am thankful to Mr. D.R. Joshi for his keen interest in typing this manuscript.

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# C O N T E N T S

| Chapter |                                                                                                                  | Pages    |
|---------|------------------------------------------------------------------------------------------------------------------|----------|
|         | Synopsis                                                                                                         | 1 - 5    |
| 1       | Optimum Runge-Kutta-Fehlberg<br>Methods for First Order<br>Differential Equations                                | 6 - 32   |
|         | 1. Introduction                                                                                                  | 6        |
|         | 2. Formula of order $h^{m+6}$                                                                                    | 6        |
|         | 3. Formula of order $h^{m+5}$                                                                                    | 15       |
|         | 4. Formula of order $h^{m+4}$                                                                                    | 18       |
|         | 5. Stability Region for<br>formula of order $h^{m+6}$                                                            | 20       |
|         | 6. Examples                                                                                                      | 22       |
|         | Tables                                                                                                           | 24       |
|         | References                                                                                                       | 31       |
| 2       | Optimum Runge-Kutta-Fehlberg<br>Methods for Second Order<br>Differential Equations                               | 33 - 73  |
|         | 1. Introduction                                                                                                  | 33       |
|         | 2. Formula of order $h^{m+6}$                                                                                    | 34       |
|         | 3. Formula of order $h^{m+5}$                                                                                    | 46       |
|         | 4. Formula of order $h^{m+4}$                                                                                    | 50       |
|         | 5. Stability Region for the<br>formula of order $h^{m+6}$                                                        | 53       |
|         | 6. Numerical Comparison                                                                                          | 60       |
|         | Tables                                                                                                           | 62       |
|         | References                                                                                                       | 72       |
| 3       | Optimum Runge-Kutta-Fehlberg<br>Methods for Simultaneous<br>Differential Equations of<br>First and Second Orders | 74 - 106 |
|         | 1. Introduction                                                                                                  | 74       |
|         | 2. Formula of order $h^{m+4}$<br>(First order)                                                                   | 75       |

|                                                                    |     |
|--------------------------------------------------------------------|-----|
| 3. Formula of order $h^{m+5}$                                      | 81  |
| 4. Formula of order $h^{m+4}$<br>(second order)                    | 90  |
| 5. Flow chart for solving<br>first order simultaneous<br>equations | 97  |
| 6. Tables                                                          | 99  |
| References                                                         | 106 |

#### 4

|                                                                                     |           |
|-------------------------------------------------------------------------------------|-----------|
| Optimum Runge-Kutta-Fehlberg<br>Methods for $n$ -th order<br>Differential Equations | 107 - 169 |
| 1. Introduction                                                                     | 107       |
| 2. Formula of order $h^{m+m+2}$                                                     | 108       |
| 3. Higher derivatives of the<br>dependent variable                                  | 110       |
| 4. Formula of order $h^{m+6}$<br>(Third order Differential<br>Equation)             | 116       |
| 5. Formula of order $h^{m+6}$<br>(Fourth order Differential<br>Equation)            | 130       |
| 6. Numerical Comparison                                                             | 145       |
| Tables                                                                              | 147       |
| References                                                                          | 169       |