

ON STATE AND PARAMETER ESTIMATION PROBLEMS
IN SOME DISTRIBUTED SYSTEMS

by

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CERTIFICATE

This is to certify that the thesis, "On State And Parameter Estimation Problems in Some Distributed Systems", which is being submitted by Amaranatha Jha for the award of degree of Doctor of Philosophy to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last three years under my supervision. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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Dedicated to my parents

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ABSTRACT

This thesis is concerned with the problem of state and parameter estimation in distributed parameter systems with noisy measurement data. In certain situations, where accessibility is restricted to the system boundary, the measurements are available only on the boundary of the system. The innovations approach has been used to obtain the Kalman-Bucy type distributed filter equations when the measurements corrupted with coloured noise are made on the boundary of the system. This approach does not require any discretization and limiting procedures. The problem of state-estimation for distributed parameter systems that have both the transportation and measurement delays has also been considered.

The finite element method has been used to approximate the distributed parameter system by a lumped system. There are two kinds of approximations carried out in the literature to get numerical solutions for estimation problems for distributed systems: approximation at the beginning or approximation at the end. It is noted that in general the state-estimation through approximation at the end would lead to complicated equations to be solved. Also in terms of the accuracy of the final results, the method does not offer any great advantages in comparison with the results obtained through discretization at the beginning.

The problem of parameter estimation in distributed system using noisy measurement is considered. The extended Kalman filter has been used to estimate the parameters after approximating the distributed system by a lumped system.

A method for estimating constant parameters of a packed bed chemical process described by partial integro-differential equation has been developed. The Laguarre polynomials have been used to estimate one of the parameters whereas the second parameter has been estimated by the combination of the extended Kalman filter and the Golden-Section Search optimization technique. The identification procedure is an iterative one.

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