# THE UNIFORM, FINE AND GRAPH TOPOLOGIES ON SPACES OF CONTINUOUS FUNCTIONS

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# THE UNIFORM, FINE AND GRAPH TOPOLOGIES ON SPACES OF CONTINUOUS FUNCTIONS

by

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*Submitted*
*in fulfillment of the requirements of the degree of*
*Doctor of Philosophy*

*to the*

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*Dedicated to*

*My Family*

# Certificate

This is to certify that the thesis entitled **Uniform, Fine and Graph Topologies on Spaces of Continuous Functions** submitted by **Mr**. **Varun Jindal** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by him under my guidance and supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

New Delhi
April 2014

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# Abstract

This thesis studies two kinds of spaces, the space $C(X,Y)$ of all continuous functions from a Tychonoff space $X$ to a metric space $Y$; and the space $H(X)$ of all self-homeomorphisms on a metric space $X$, with the uniform, fine and graph topologies.

For a Tychonoff space $X$ and a metric space $(Y,d)$, the uniform, fine and graph topologies on the space $C(X,Y)$ are denoted respectively by the letters $d, f$ and $g$. The spaces $C(X,Y)$ equipped with the uniform topology $d$, fine topology $f$ and graph topology $g$ are denoted respectively by $C_d(X,Y)$, $C_f(X,Y)$ and $C_g(X,Y)$. For a metric space $(X,d)$, the fine and graph topologies on the space $H(X)$ are equal. So on the space $H(X)$, we consider only the uniform and fine topologies. The spaces $H(X)$ equipped with the uniform topology $d$ and fine topology $f$ are denoted respectively by $H_d(X)$ and $H_f(X)$. After giving definitions of the uniform, fine and graph topologies on $C(X,Y)$, we compare these topologies on $C(X,Y)$ with the well-known topologies on $C(X,Y)$ such as the point-open topology $p$ and the compact-open topology $k$ on $C(X,Y)$. Then we study the following topological properties of $C(X,Y)$ under the aforesaid topologies in terms of the topological properties of $X$:

- For $C_f(X,Y)$ and $C_g(X,Y)$

  $(i)$ Metrizability.

(*ii*) Various kinds of countability properties such as first countability, separability, second countability, Lindelöf condition and countable chain condition (ccc).

(*iii*) Various kinds of completeness properties such as complete metrizability, $\check{C}$ech-completeness, local $\check{C}$ech-completeness, sieve-completeness, partition-completeness, pseudocompleteness and the property of being a Baire space, and

(*iv*) Connectedness and path connectedness.

- Connectedness and path connectedness of the space $C_d(X, Y)$
- Metrizability, first countability, separability, connectedness and path connectedness of the space $H(X)$ under the uniform and fine topologies.

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