

Unit Groups of Group Algebras

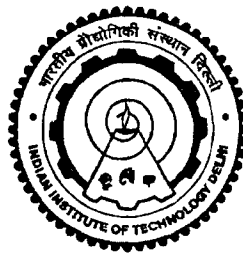
By

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*Submitted in fulfillment of the requirements
of the degree of Doctor of Philosophy*

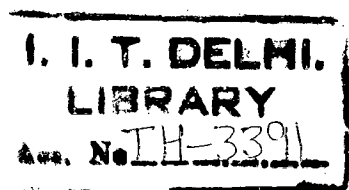
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Certificate

This is to certify that the thesis entitled " Unit Groups of Group Algebras" submitted by *Ms. Manju Khan* to the Indian Institute of Technology Delhi, for the award of the Degree of Doctor of Philosophy, is a record of bona fide research work carried out by her under our supervision. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results obtained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.

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Abstract

The aim of the thesis is to study the unit group of group algebras over fields of positive characteristic. Complete characterizations of the unit groups of group algebras of symmetric group S_n of degree n , for $n = 3, 4$ and of the alternating group A_4 of degree 4 over arbitrary finite fields have been obtained. The Lie properties of a ring R play an important role in the study of the structure of the unit group $\mathcal{U}(R)$. A new Lie augmentation technique has been developed to study certain subgroups of $\mathcal{U}(R)$. For a Lie ideal U of a ring R , $T(U) = \{a \in R \mid [a, R] \in U\}$ is a Lie ideal as well as subring of R . Define $\Lambda(U) = T(U) \cap \mathcal{U}(R)$. This is a subgroup of $\mathcal{U}(R)$. A relationship between the terms of derived series of the unit group $\mathcal{U}(R)$ and the subgroup $\Lambda(U)$, where U are the terms of lower central chain of the associated Lie ring $\mathcal{L}(R)$. The lower central chain of the associated group algebra of a dihedral group over a field has been characterized.

Contents

Certificate	i
Acknowledgements	iii
Abstract	v
1 Introduction	1
1.1 Preliminaries	2
1.2 Group Algebras	5
1.3 Associated Lie Algebras of Group Algebras	8
1.4 Unit Group	10
2 The Unit Group of FS_3	13
2.1 The Unit Group of FS_3	14
2.2 Unit Groups of $\mathbb{Z}_2S_3$ and $\mathbb{Z}_3S_3$	21
3 The Unit Group of FA_4	31
3.1 Introduction	31
3.2 Unit Group of FA_4	33
3.3 Unit Group of $\mathbb{Z}_2A_4$	39

4	The Unit Group of FS_4	41
4.1	Introduction	41
4.2	The Unit Group of $FS_4 : \text{char} F = 2$	43
4.3	The Unit Group of $FS_4 : \text{char} F = 3$	47
4.4	The Unit Group of $FS_4 : \text{char} F = p > 3$	51
5	Lie Augmentation Ideals of a Ring	57
5.1	Preliminaries	58
5.2	The Lie Augmentation Ideal	60
5.3	The Dihedral Group Algebra	67
	Bibliography	79
	Notation	85
	Bio-Data	87