ANALYTICAL STUDY OF NONLINEAR STOCHASTIC EVOLUTIONARY p-LAPLACE TYPE EQUATIONS DRIVEN BY LÉVY NOISE

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by

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Dedicated to My Family

Certificate

This is to certify that the thesis entitled **Analytical Study of Nonlinear Stochastic Evolutionary p-Laplace type equations driven by Lévy noise** submitted by **Mr. Kavin R** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy**, is a record of the original bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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Abstract

In this thesis, our focuses on examining the impacts of stochastic forcing on solutions of nonlinear partial differential equations (PDEs) through analytical methods. In particular, our research goals are directed towards analyzing the effect of perturbation by noise of nonlinear evolutionary p-Laplace type Dirichlet problems for PDEs. In this thesis, we have mainly worked on stochastic partial differential equations (SPDEs) perturbed by multiplicative Lévy noise, where the nonlinear operator (in divergence form) satisfies p-type growth with coercivity assumptions. We have answered various analytical questions related to well-posedness (existence and uniqueness), associated control problems, and some of the important probabilistic properties of the solutions, namely, Freidlin-Wentzell's large deviation principle (LDP), quadratic transportation cost inequality (TCI), and existence of invariant measure.

In Chapter 1, we provide fundamental definitions and available results. Moreover, we present an overview of the thesis's organization and highlights our essential contributions. In Chapter 2, we establish a large deviation principle for the strong solution (in probabilistic sense) of an evolutionary p-Laplace equation driven by small multiplicative Brownian noise, where the weak convergence approach plays a key role. Moreover, by using Girsanov transformation along with the L^1-contraction approach, we show the quadratic transportation cost inequality for the strong solution to the underlying problem.

In Chapter 3, we study a nonlinear stochastic control problem perturbed by multiplicative Lévy noise, where the nonlinear operator (in divergence form) satisfies p-type growth with coercivity assumptions. By using Aldous tightness criteria and Jakubowski's version of the Skorokhod theorem on non-metric space along with the standard L^1-method, we establish the existence of a path-wise unique strong solution. Formulating the associated control problem, and using a variational approach together with the convexity property of the cost functional (in control variable), we establish the existence of a weak optimal solution to the underlying problem. We use the technique of Maslowski and Seidler to prove the existence of an invariant measure for the uncontrolled SPDEs driven with multiplicative Lévy noise.

In Chapter 4, we explore Freidlin-Wentzell's large deviation principle for a stochastic partial differential equation with the nonlinear diffusion-convection operator in divergence form satisfying p-type growth, involving coercivity assumptions, perturbed by small multiplicative Brownian noise. We use the weak convergence method to prove the Laplace principle, which is equivalent to the large deviation principle in our framework.

In Chapter 5, we study the well-posedness theory for solutions of the stochastic heat equations with logarithmic nonlinearity perturbed by multiplicative Lévy noise. By using Aldous tightness criteria and Jakubowski's version of the Skorokhod theorem on non-metric spaces along with the standard L^2-method, we establish the existence of a pathwise unique strong solution. Moreover, by using a weak convergence method, we establish a large deviation principle for the strong solution of the underlying problem. Due to the lack of linear growth and locally Lipschitzness of the term $u\log(|u|)$ present in the underlying problem, the logarithmic Sobolev inequality and the nonlinear versions of Gronwall's inequalities play a crucial role.

सार

इस शोध प्रबंध में, हमारा ध्यान विश्लेषणात्मक तरीकों के माध्यम से गैर-रेखीय आंशिक विभेदक समीकरणों (PDEs) के समाधानों पर स्टोकेस्टिक फोर्सिंग के प्रभावों की जांच करने पर केंद्रित है। विशेष रूप से, हमारा शोध लक्ष्य गैर-रेखीय विकासात्मक p-लाप्लास प्रकार के डाइरिचलेट समस्याओं पर शोर द्वारा बाधा के प्रभाव का विश्लेषण करना है। इस शोध प्रबंध में, हमने मुख्य रूप से बहुगुणक लेवी शोर से प्रभावित स्टोकेस्टिक आंशिक विभेदक समीकरणों (SPDEs) पर काम किया है, जहाँ गैर-रेखीय ऑपरेटर (विभाजन रूप में) p -प्रकार की वृद्धि और जबरदस्ती (coercivity) मान्यताओं को संतुष्ट करता है। हमने समाधान की अच्छी-परिभाषिता (अस्तित्व और अद्वितीयता), संबंधित नियंत्रण समस्याओं, और समाधानों के कुछ महत्वपूर्ण प्रायिक गुणधर्मों जैसे कि फ्राइडलिन-वेंट्ज़ेल का बड़ा विचलन सिद्धांत (LDP), द्विघात परिवहन लागत असमानता (TCI), और अपरिवर्तक माप (invariant measure) की उपस्थिति से जुड़े विभिन्न विश्लेषणात्मक प्रश्नों के उत्तर दिए हैं।

अध्याय 1 में, हमने बुनियादी परिभाषाएँ और उपलब्ध परिणाम प्रस्तुत किए हैं। इसके अतिरिक्त, हमने शोध प्रबंध की संरचना का अवलोकन प्रस्तुत किया है और हमारे मुख्य योगदानों को रेखांकित किया है।

अध्याय 2 में, हमने छोटे बहुगुणक ब्राउनियन शोर से प्रेरित विकासात्मक p-लाप्लास समीकरण के मजबूत समाधान (प्रायिक अर्थ में) के लिए बड़ा विचलन सिद्धांत स्थापित किया है, जहाँ कमजोर अभिसरण विधि एक महत्वपूर्ण भूमिका निभाती है। इसके अलावा, गिर्सानोव परिवर्तन और L^1-संकुचन दृष्टिकोण का उपयोग करते हुए, हमने अंतर्निहित समस्या के मजबूत समाधान के लिए द्विघात परिवहन लागत असमानता को प्रदर्शित किया है।

अध्याय 3 में, हमने बहुगुणक लेवी शोर से प्रभावित गैर-रेखीय स्टोकेस्टिक नियंत्रण समस्या का अध्ययन किया है, जहाँ गैर-रेखीय ऑपरेटर (विभाजन रूप में) p-प्रकार की वृद्धि और जबरदस्ती मान्यताओं को संतुष्ट करता है। एल्डस की संकुचन कसौटी और नॉन-मैट्रिक स्पेस पर स्कोरोखोड प्रमेय के जाकूबोव्स्की संस्करण के साथ मानक L^1-विधि का उपयोग करते हुए, हमने एक पथ-वार अद्वितीय मजबूत समाधान के अस्तित्व को स्थापित किया है। संबंधित नियंत्रण समस्या को तैयार करते हुए, और लागत फलन की उत्तलता गुणधर्म के साथ विविधतापूर्ण दृष्टिकोण का उपयोग करते हुए, हमने अंतर्निहित समस्या के लिए कमजोर इष्टतम समाधान के अस्तित्व को स्थापित किया है। अनियंत्रित SPDEs के लिए अपरिवर्तक माप के अस्तित्व को साबित करने हेतु, हमने मासलोव्स्की और सेडलर की तकनीक का उपयोग किया है।

अध्याय 4 में, हमने गैर-रेखीय विसरण-संवहन ऑपरेटर के साथ स्टोकेस्टिक आंशिक अंतर समीकरणों के लिए फ्राइडलिन-वेंट्ज़ेल के बड़े विचलन सिद्धांत की खोज की है, जो p-प्रकार की वृद्धि और जबरदस्ती मान्यताओं को संतुष्ट करता है, और छोटे बहुगुणक ब्राउनियन शोर से बाधित है। हमने कमजोर अभिसरण विधि का उपयोग करके लैप्लास सिद्धांत को साबित किया है, जो हमारे ढाँचे में बड़े विचलन सिद्धांत के समतुल्य है।

अध्याय 5 में, हमने बहुगुणक लेवी शोर से बाधित लॉगरिदमिक गैर-रेखीयता के साथ स्टोकेस्टिक हीट समीकरणों के समाधान के लिए अच्छी-परिभाषिता सिद्धांत का अध्ययन किया है। एल्डस की संकुचन कसौटी, गैर-मैट्रिक स्थानों पर स्कोरोखोड प्रमेय के जाकूबोव्स्की संस्करण, और मानक L^2-विधि का उपयोग करके, हमने एक पथ-वार अद्वितीय मजबूत समाधान के अस्तित्व को स्थापित किया है। इसके अलावा, हमने कमजोर अभिसरण विधि का उपयोग करके अंतर्निहित समस्या के मजबूत समाधान के लिए बड़े विचलन सिद्धांत को स्थापित किया है। चूँकि अंतर्निहित समस्या में $u \log(|u|)$ शब्द के रेखीय वृद्धि और स्थानीय रूप से लिप्शिट्ज़ गुणधर्म की कमी है, इसलिए लॉगरिदमिक सोबोलेव असमानता और ग्रोनवाल की असमानताओं के गैर-रेखीय संस्करण एक महत्वपूर्ण भूमिका निभाते हैं।

Contents

List of Symbols

Symbol	Meaning
τ	Time discretization parameter
$\mathbb{N}$	Set of natural numbers
$\mathbb{R}$	Set of real numbers
$\|u\|_X$	Norm of u in normed linear space X
D	$D \subset \mathbb{R}^d$ and bounded domain
∂D	Lipschitz boundary for domain D
p'	Convex conjugate of p, $1 \leq p < \infty$
L^p	Lebesgue space on D, $1 \leq p < \infty$
$W_0^{1,p}$	Sobolev space on D, $1 \leq p < \infty$
$W^{-1,p'}$	Dual space of $W_0^{1,p}$
$W^{\alpha,p}$	Fractional Sobolev space, $\alpha \in (0,1)$, $1 < p < \infty$
$D_{(0,T)}$	$(0,T) \times D$ for fixed $T > 0$
$\partial D_{(0,T)}$	$(0,T) \times \partial D$
$(\mathbb{X}, w)$	Topological space $\mathbb{X}$ equipped with the weak topology
$\mathcal{L}_2(H;K)$	Space of Hilbert-Schmidt operators from Hilbert space H into Hilbert space K
$\mathcal{L}_2(H)$	Space of Hilbert-Schmidt operators from H into H
$(\cdot,\cdot)_{L^2}$	Scalar product on L^2
$\langle \cdot,\cdot \rangle_{W^{-1,p'},W_0^{1,p}}$	Dualization between $W^{-1,p'}$ and $W_0^{1,p}$