

ON SPEED AND TORQUE REGULATION OF INDUCTION MOTORS  
USING LINEAR QUADRATIC REGULATION THEORY

by

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TO  
MY BELOVED DAUGHTER  
RIVA

## PREFACE

The thesis is based on the results of the author's independent investigations in the Department of Electrical Engineering, Indian Institute of Technology, Delhi, carried out during the period Sept'72 - Feb'76. The main objective of these investigations was to obtain feedback controller structures for regulating the speed and torque of a three-phase slip ring induction machine by making use of the linear quadratic regulator theory. While, it seemed possible to develop a state variable model for the machine directly, it was found analytically more convenient to apply a d-q axes transformation. This transformation, in effect, reduces the dimensions of state and control variables and also converts the time varying model of the actual machine into an time invariant equivalent model. However, the output variables namely speed and torque of the equivalent machine are the same as those of the actual machine, so that the performance analysis in terms of equivalent machine is indicative of the performance of the actual machine also. Once the results are obtained on the basis of the time invariant model, scheme of implementation of the control solution on the actual machine using time varying feedback gains can be developed.

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Raj Kumar Gupta

## A B S T R A C T

This thesis describes the results of the candidate's investigation on the problem of regulator design for speed and torque controls of three-phase slip-ring induction motors. It is proposed that feedback regulators for such machines can be built by making use of the linear quadratic regulator theory. The various results reported are spread over six chapters whose contents are as follows.

The first chapter is an introductory chapter. It starts with a discussion on the problem of setting up of a suitable state variable model of the three-phase slip-ring induction motor. It is shown that starting from the voltage-current relations available in the literature, time invariant, linear state model can be developed if one takes the synchronously rotating d-q axes as the frame of reference. The second part of this chapter is concerned with statements of the various situations dealt with in the rest of the thesis.

In chapter-2 we deal with the problem of experimentally verifying the theoretically derived state model of a specific machine. A common criticism of results based on optimal control theory is that the basic model involves a lot of idealisations. The state model derived in Chapter-1 is, of course, based on several simplifying assumptions. It was, therefore, felt necessary to check the validity of the model

through experimentation. Results of parameter estimation through Kalman-filtering techniques for a specific machine are discussed.

The problem of speed regulation of the machine studied in chapter-2 is discussed in chapter-3. Constant-gain state feedback schemes for maintaining the speed at a preassigned value is developed by making use of the infinite time linear quadratic regulator theory. The proposed scheme is tested for the experimental machine by simulating it on a digital computer. Effects of changing the weighting matrices of the cost-function on the transient response of the controlled system are also illustrated.

Chapter-4 is concerned with the problem of torque-regulation of the same machine. Feedback control schemes are again developed on the basis of infinite time linear quadratic regulator theory. Results are also presented on the basis of simulation studies.

Chapter-5 presents the modifications necessary in the proposed control schemes in order to take care of unknown but constant disturbances in the system. Proportional state and integral output feedback schemes for the machine under study are developed on the basis of Smith and Davison's approach. The results are illustrated by simulation studies for both the speed and torque control cases.

In chapter-6, we study the modifications necessary in the control scheme for achieving control by feeding back the output alone. We make use of the technique of Kosut to develop desired output feedback controllers. The results are again illustrated through simulation studies.

The main results are summarised in chapter-7, where some suggestions for further studies including the possible scheme of implementation of the proposed control schemes are given.

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