

VIBRATION RESPONSE OF MULTILAYER BEAMS

SUBJECTED TO SHOCK EXCITATION

by

Arjun Dev Kapur

Thesis submitted to the Indian Institute of Technology,
Delhi, for the award of the degree of

DOCTOR OF PHILOSOPHY

Department of Mechanical Engineering,
Indian Institute of Technology, Delhi.

AUGUST, 1974

ABSTRACT

A study of flexural vibrations of various multilayer beam configurations, subjected to shock excitation in the form of half-sine wave pulse acceleration, has been carried out for simply supported as well as cantilever type end conditions.

In the case of simply supported beams, the configurations studied are 5-layer beams with viscoelastic cores, 3-layer beams with elastic or viscoelastic cores and 2-layer beams with one of the layers as viscoelastic. Equations of motion and associate terminal conditions are firstly derived for a 5-layer beam subjected to shock excitation. These very equations of motion and terminal conditions are then utilised in deducing the governing equations and boundary conditions for all the remaining cases mentioned above. The derivations take into account rotary and longitudinal inertias besides transverse inertia for all the layers. The other effects included are the shear strain energy of the cores and extensional and bending strain energy of the elastic facings and unconstrained viscoelastic layers. Properties of the viscoelastic material in shear strains or in direct strains are represented by a 4-element viscoelastic model throughout. Series solutions, satisfying the end conditions, are assumed for the displacement components. Expressions for the shock response of displacement components and surface strain are worked out for the above multilayer arrangements. Effects of rotary and longitudinal inertias and other system

parameters on the shock response effectiveness of all such arrangements are reported and discussed. Also, the shock response effectiveness of a 3-layer undamped beam is compared with the corresponding effectiveness of a homogeneous beam and that of a 3-layer damped beam on the basis of certain specified criteria.

In the case of cantilever type beams, the work is restricted to the study of the shock response of 2 and 3-layer configurations, subjected to half-sine shock loading, taking only transverse inertia into consideration. A 4-element viscoelastic model is used to characterise the properties of the viscoelastic material. Solution for the transient response of a 2-layer cantilever, with one of the layers as viscoelastic, is obtained by the separation of variables technique. Influence of various geometrical and physical parameters on the shock response is studied. The problem of shock response of a 3-layer cantilever with elastic or viscoelastic core is dealt with, using an approximate variational approach based on Kantorovich Method. Expressions for the transient displacement and strain response are worked out and limitations of the approximate approach are indicated.

The shock response effectiveness of various simply supported beams is then compared for different criteria on constant size-and-weight basis. Further, experimental work is carried out on a few samples of multilayer beams. The test results in respect of transient strain response are compared with the

corresponding theoretical values to check the validity of the analytical methods employed. Finally, the general discussion and conclusions about the present work and the scope for the future work are given at the end.

CERTIFICATE

This is to certify that the Thesis entitled "Vibration Response of Multilayer Beams subjected to Shock Excitation" being submitted by Mr. Arjun Dev Kapur to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mechanical Engineering, is a record of bonafide research work carried out by him. He has worked under my guidance and supervision and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



(B.C. NAKRA)
Professor,
Mechanical Engineering Deptt.,
Indian Institute of Technology,
DELHI

ACKNOWLEDGEMENTS

The author wishes to express his deep sense of gratitude to Dr. B.C. Nakra for suggesting the problem and for providing valuable and inspiring guidance. His continuous encouragement made the task lighter.

Thanks are due to Mr. R.P. Mahna of the 'Vibration and Instrumentation' Laboratory, for his unstinted help in the experimental work. Thanks are also due to the staff of the Computer Centre for their cooperation in the computation work.

Financial support for the present work was made available under a centrally sponsored scheme (Q.I.P.) by the Ministry of Education, Govt. of India. This is gratefully acknowledged.

Finally, the author presents his hearty thanks and sincere apologies to his near and dear ones, who very willingly endured certain hardships which resulted from author's preoccupation with the present work.



(Arjun Dev Kapur)

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