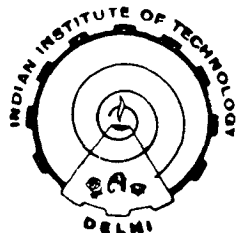


# **SAMPLING DESIGNS FOR SIGNAL DETECTION AND ESTIMATION**

By  
**MIRZA. M. WAHAJ**

A thesis submitted to  
Indian Institute of Technology, Delhi  
for the award of the degree of  
**DOCTOR OF PHILOSOPHY**  
in  
**Electrical Engineering**



Department of Electrical Engineering  
**INDIAN INSTITUTE OF TECHNOLOGY, DELHI**

June 1989


**DEDICATED**  
**TO**  
**MY PARENTS**

## CERTIFICATE

This is to certify that the thesis, entitled "SAMPLING DESIGNS FOR SIGNAL DETECTION AND ESTIMATION", submitted by MIRZA.M.WAHAJ for the award of the degree of Doctor of Philosophy to the Indian Institute of Technology is a record of the bonafide research work carried out by him under our joint supervision form July 1984 to April 1989. The results contained in this thesis have not, been submitted to any other University or Institute for the award of any degree or diploma.



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**Mirza Mohd. Wahaj**

## **Abstract**

This dissertation deals with specific signal detection and estimation problems in the context of random and deterministic sampling designs in one and two dimensions.

The problem of detecting known binary signals in the presence of additive gaussian noise is investigated in the context of simple random sampling first. Using simple detector coefficients, the simple random sampling design leading to maximum signal to noise ratio has been determined. Further, in this context, the optimum signal set as well as the least favorable signal and the corresponding sampling design have been found for the general binary detection. The performance of the random sampling scheme has been compared to those of the optimal continuous time detection and discrete time detection based on uniform sampling.

The problem of detecting signals with random phase is next formulated in a general set up and the sampling design yielding maximum probability of detection at a given false alarm rate is then determined using an approximation procedure. The performance of this sampling design is compared to that of the corresponding optimal continuous time detector and the finite sample detector based on uniform sampling in a stationary Gauss–Markov noise environment.

The linear minimum mean square signal estimation problem is investigated in the context of random and deterministic sampling designs next. Using simple estimator coefficients, a simple random sampling design which results in minimum mean square estimation error between the signal and its estimate is determined. In the context of deterministic sampling designs, using simple and optimal estimator coefficients, midpoint sampling designs which yield minimum mean square error between the signal and its estimate, have been determined and their performances have been compared to those of

the random sampling design, the optimal continuous time estimators and the ones based on uniform sampling.

Finally, the problem of image restoration by two-dimensional Wiener filtering has been investigated in the context of a spatial simple random sampling design. The spatial sampling design yielding minimum mean square restoration error between an ideal image and its restored replica at the output of the two-dimensional Wiener filter has been determined and its performance has been compared to that of the corresponding optimal continuous data estimator. An iterative solution for the two-dimensional integral equation characterizing the image restoration problem by two-dimensional Wiener filtering has also been discussed.

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