

SOME FAST COMPUTATIONAL ALGORITHMS FOR
SIGNAL PROCESSING

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CERTIFICATE

This is to certify that the thesis entitled, ' SOME FAST COMPUTATIONAL ALGORITHMS FOR SIGNAL PROCESSING ' being submitted by Sudhakar, R., to the Department of Electrical Engineering, Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him under our supervision and guidance and in our opinion, it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted to any other institute for the award of any degree or diploma.



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ABSTRACT

This thesis presents some new and efficient algorithms for solving the following important digital signal processing problems:

- (i) fast computation of Fourier transform at arbitrary frequencies;
- (ii) efficient time domain interpolation at arbitrary instants;
- (iii) frequency estimation in presence of broad-band noise; and
- (iv) frequency estimation in presence of other sinusoids. The computational advantages of the proposed algorithms arise from some simplifying approximations introduced at various stages of solution and it is proved that the resulting errors lie within acceptable bounds in the usual applications.

A computationally superior scheme is presented for the evaluation of Fourier transform of a discrete time sequence at arbitrary frequencies by interpolating the discrete Fourier transform samples. It is shown that the decaying nature of the interpolating function can be exploited to approximate it with the low order terms of its Taylor series expansion, except in the neighborhood of its narrow peak. The approximated part is then evaluated as a cyclic convolution, and over the peaky region, the function is directly computed. A conceptually similar method is developed for interpolating a band-limited signal at arbitrary time instants from its uniform time samples. The Taylor series approximation results in the need of discrete differentiations of the signal sequence. Approximate differentiators having simple binary coefficients and requiring no

multiplications, are proposed to reduce the computation.

A related class of problems, viz., that of estimating the frequency of a sinusoid in noisy environment is investigated. Two methods are suggested, the applicability of one or the other depending upon whether the noise is broad-band or narrow-band. In these methods, the signal sequences are subjected to exclusive time domain processing, and the sinusoid frequency is estimated through a modified zero crossing counting operation on the processed signals. When the noise is broad-band, the sinusoidal component is enhanced by repeated autocorrelation operations. By resorting to coarse quantization of the sequences and then performing the autocorrelations, substantial savings in computation is achieved; the loss in the usable range of signal-to-noise ratio due to these quantizations, is shown to be relatively insignificant. When the interfering signal contains only a few other sinusoids whose possible frequencies are known beforehand, the processing consists of attenuating the undesired sinusoids sufficiently, by using a cascaded combination of simple, finite impulse response null filters. These null filters are shown to require only a few additions and thus help to achieve reduction in computation.

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