A Numerical Investigation of Natural Convection in a Horizontal Rotating Cylinder

by

Nadeem Hasan

Department of Applied Mechanics

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the

Indian Institute of Technology, Delhi

INDIA

October 2005

Cylinder
Rotating cylinder

Certificate

This is to certify that the thesis entitled **A Numerical Investigation of Natural Convection in a Horizontal Rotating Cylinder** being submitted by **Nadeem Hasan** to the **Indian Institute of Technology, New Delhi (India)** for the award of the degree of Doctor of Philosophy in Applied Mechanics Department is a bonafide research work carried out by him under my supervision and guidance. The research reports and the results presented in this thesis have not been submitted in parts or in full to any other University or Institute for the award of any degree or diploma.

Sanjeev Sanghi

Sanjeev Sanghi, Ph. D.

Associate Professor
Applied Mechanics Department
Indian Institute of Technology
New Delhi-110016

Acknowledgements

I would like to take this opportunity to thank those who helped and supported me throughout this effort. In the capacity of my research supervisor, Dr. Sanjeev Sanghi was always available for guidance and gave me total freedom to experiment with different ideas. This work evolved through many fruitful discussions with him. The presentation of the work has been largely shaped by his expertise and experience. I would also like to thank Dr. M. R. Ravi and Dr. Krishnan. V. Pagalthivarthi for being part of my research committee. Their critical assessment and invaluable suggestions have helped me in improving the overall quality of the work.

There have been many others who have contributed towards this work. Specifically, I would like to thank Mr. Kailash Chand for his help and cooperation in carrying out the numerical simulations in the computational laboratory. I am also grateful to a number of colleagues for their assistance in the compilation of the thesis.

I would also like to take this opportunity to thank my family members for their cooperation and the support, especially in some difficult and testing times. Thanks to my wife for standing by me in some difficult phases and, shouldering a lot of responsibilities of the family, thereby allowing me to concentrate on the research. Above all, thanks to the Almighty for showing me the light in moments of darkness.

Nadeem Hasan

Abstract

The dynamics of the two–dimensional thermally driven flow in a horizontal rotating cylinder subjected to a periodic temperature perturbation in the circumferential direction has been investigated numerically. The problem has been formulated for a Cartesian rotating frame of reference subject to the Boussinesq approximation. Starting from a state of solid–body rotation or a state of rest with respect to the rotating observer, the flow is initiated under the combined action of a time–periodic gravitational buoyancy force and a centrifugal or rotational buoyancy force. A semi-explicit pressure correction scheme with a finite difference type of discretization, on a structured colocated Cartesian mesh, has been employed for the purpose of the numerical study.

The dimensionless forms of the governing equations, in the non-inertial rotating Cartesian frame of reference, reveal the presence of four dimensionless groups or parameters relevant to the problem. These are, 1) Gravitational Rayleigh number (Ra_g), 2) Rotational Rayleigh number (Ra_Ω), 3) Taylor number (Ta) and 4) Prandtl number (Pr). In order to obtain physically realizable solutions, the ratio Ta / Ra_Ω {= 1 / (β ΔT Pr)} is kept fixed at 100, while the dynamics is explored in a parametric space of (Ra_g, Ra_Ω, Pr). At a fixed Pr and Ra_g, the numerical simulations are carried out over a wide range of Ra_Ω spanning the interval [$10^2 - 10^7$]. Three values of Ra_g given as 10^4, 10^5 and 10^6 have been considered. The simulations are carried out for Pr = 0.71 as well as Pr = 7.0. The study is primarily aimed at investigating the complex unsteady flow dynamics and its impact on the heat transfer characteristics.

Contrary to the ideas expressed in some earlier studies regarding the role of the coriolis force in two-dimensional thermally driven flows in rotating square enclosures, it is shown theoretically and demonstrated through numerical experiments

that, within the framework of two-dimensionality and the Boussinesq approximation, the coriolis force is irrotational in character and does not affect the evolution of the velocity and the temperature fields in such class of flows.

Over the entire range of solutions for each Prandtl number, the unsteady dynamics is investigated by identifying the different flow regimes dominated by the different body forces. In the flow regime controlled by gravitational buoyancy (Regime I), the flows mostly exhibit a periodic character with frequency locked to the rotation frequency of the gravity vector. A few quasi-periodic solutions are also found to exist in this regime. The solutions in Regime I are primarily controlled by the parameter $\left(\frac{\mathrm{Ta.Pr}}{\mathrm{Ra}_g}\right)^{1/2}$ which reflects the ratio of the driving frequency of the periodic gravitational buoyancy force to the natural frequency of convective motion of the buoyant fluid elements around the circular enclosure. Depending on the values of this parameter, the unsteady flows can exhibit unidirectional clockwise circulatory flow patterns as well as patterns that exhibit changes from clockwise to counterclockwise circulatory flows over one rotation cycle of the gravity vector. In Regime II, the role of the main driving force is exchanged between the gravitational and the centrifugal buoyancy and the resulting flow patterns are governed by the actual balance between the two forces. The convection in this regime is very much suppressed. The fluctuation levels are low and the flows exhibit periodicity with frequency–locking as well as quasi–periodicity. The flows in Regime III are dominated by centrifugal buoyancy and are characterized by flow patterns comprising of two rolls aligned along the horizontal diameter (in the rotating frame). The flows exhibit periodicity with frequency–locking as well as quasi–periodicity depending upon the values of the various dimensionless parameters. The increase in Ra_Ω induces bifurcations causing the quasi–periodicity which culminates in a highly complex multi-cellular flow pattern in space and irregular high frequency oscillations in time as Ra_Ω approaches a value of 10^7.

As an impact of the spatiotemporal dynamics in the three flow regimes, the time-averaged heat transfer characteristics for the heated portion of the enclosure wall, exhibit a non-monotonic variation in Regime I and Regime II with increase in Ra_Ω for the different values of Ra_g and Pr. In Regime III the mean heat transfer is virtually independent of Ra_g and increases monotonically with Ra_Ω. Increase in Pr from 0.71 to 7.0 affects the mean heat transfer in Regime I and Regime II significantly by not only shifting the characteristics towards lower values of Ra_Ω but also altering the form of the characteristics. The increase in Pr has only a slight effect on the characteristics in Regime III.

The POD analysis of the unsteady flow field at $Ra_g = 10^5$ and $Pr = 0.71$ reveal changes in the dominant spatial structures as captured by the leading modes or the eigenfunctions. A new criterion of error measure for the reconstructed flow field using the POD modes is proposed. This criterion takes into account the different convergence rates of the components of a composite vector field comprising of flow velocities and scalars like the temperature field and reflects directly the accuracy with which a given data ensemble can be reconstructed using a finite number of modes. With the exception of the case of $Ra_\Omega = 10^7$, not more than 20 modes are needed to capture the unsteady flow fields within 5% accuracy of the data. At $Ra_\Omega = 10^7$, a large number of modes (>120) are needed to accurately capture the spatially complex multi-cellular flow with irregular high frequency oscillations in time. Some propagating structures are also observed in the flows at $Ra_\Omega = 10^3$ and $Ra_\Omega = 10^6$. The low–dimensional models, constructed through Galerkin projection of the POD modes onto the equations governing the fluctuating flow field, show that the temporal behaviour of the system is analogous to that of a periodically forced non-linear oscillator. It is shown that at most a 20 dimensional model is needed to capture the periodic and the quasi–periodic behaviour of the flow over the range $10^2 \leq Ra_\Omega \leq 10^6$.

Contents