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THESIS

# AXISYMMETRIC FLOW AND HEAT TRANSFER IN HYDRODYNAMICS AND MAGNETOHYDRODYNAMICS



BY

Manjit Singh Jawa
Junior Research Fellow (C.S.I.R.)
Department of Mathematics
Indian Institute of Technology, Delhi
New Delhi-29
INDIA.

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M.S. Jana 10.10.66

( M.S. JANA )

ndian Institute of
echnology, Delhi.

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CERTIFICATE

This is to certify that the thesis entitled 'Axisymmetric Flow and Heat Transfer in Hydrodynamics and Magnetohydrodynamics' which is being submitted by Mr. Manjit Singh Jawa for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi is a record of bonafide research work. He has worked for the last two years under my guidance and supervision after getting his Post-M.A., Diploma (D.I.I.T) in Rheology and Plasticity from Indian Institute of Technology, Kharagpur.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

M.K. Jain

( M.K. Jain )
Head of the Department of Mathematics
Indian Institute of Technology
Delhi.

## SYNOPSIS

Since the beginning of twentieth century there has been a great increase of knowledge pertaining to the mechanics of fluids, and during this period some of the progress in all branches of engineering has been founded on, or assisted by a better understanding of fluid flow.

The nature of fluid flow around and between rotating and oscillating axially symmetric bodies became the subject matter of investigations in the first place in connection with designing of viscometers and turbo-machinery. During the last quarter century an unprecedented development, and an increase in the field of application of turbomachinery for the compression of gases and vapours has taken place. The plane surface, perpendicular to the axis of rotation of oscillation, forms an essential part of rotors of turbines and of viscometers, and therefore, the analysis of flows involved in such cases is still a very rewarding field for research. The determination of the viscous resistance to the movement of disks is essential in estimating the power expended in the rotation of turbines and calculation of viscosity etc.

The construction of gas turbines, and the use of high temperature steam in 'steam turbines' has led to the question of measuring the heat loss due to rotating elements

of turbines particularly from disks and cylinders. This knowledge is also required in the development of methods of artificial cooling. The flight of rotating shells, the theory of sliding bearings, etc., are few other domains of technology where measurements of the resistance and the heat transfer are of great value.

More recently, it has been recognised that combination of fluid motion and electromagnetic effects can lead to much more varied phenomena than either one alone; as such it has become customary to extend the problems of Hydrodynamics to Magnetohydrodynamics. The later discipline has more than proved its worth in the problems of missile and spacecraft dynamics, propulsions and communications.

This thesis entitled "Axisymmetric Flow and Heat Transfer in Hydrodynamics and Magnetohydrodynamics" presents theoretical analysis of flow patterns, viscous resistance at the walls and heat propagation throughout the region under consideration. The effects of porosity have also been studied. Numerical results have been tabulated and their graphical representations are given at the end of each chapter.

The thesis has been divided into six chapters. The first chapter deals with the basic equations governing the flow and heat transfer phenomena. A critical survey of the relevant literature has also been made and a brief account of the problems discussed in the thesis and of the methods of solutions is given.

In the second chapter different aspects of 'Hydromagnetic Forced Flow against a Rotating Disk' are discussed. An exact numerical solution of the resulting non-linear differential equations has been obtained by the Runge-Kutta method of integration. It has been found that the flow velocity and rate of heat transfer are decreased by the increase in the strength of magnetic field. The increase in the forced flow parameter increases the radial flow and the rate of heat transfer. The results obtained in this chapter are in good agreement with the results obtained by other authors in the particular cases which arise out of the present problem by putting, (i) forced flow parameter equal to zero, (ii) hydromagnetic interaction parameter equal to zero and (iii) by putting both the parameters equal to zero.

The third chapter deals with the problem of 'Flow of a Viscous Liquid Between a Torsionally Oscillating Disk and a Stationary Porous Disk.' The resulting set of non-linear differential equations is solved by developing a regular perturbation scheme for velocity functions and pressure constant. The investigations reveal that porosity has great bearings on the steady components of radial velocity, axial velocity and radial shearing stresses. For example, when the stationary disk is non-porous, the radial flow is outward near the oscillating disk, inward near the stationary disk and vanishes about half way between the disks. The flow is outward near both disks when the porous disk sucks fluid and it is inward near both the disks when fluid is ejected through the disk; but in both the situations the radial velocity is maximum about half way between the disks.

An investigation of 'The Axially Symmetric Flow of a Viscous Liquid and Heat Transfer Between Two Infinite Porous Slowly Rotating Disks' has been made in the fourth chapter. It is observed that twenty seven different physical situations can arise in this problem, and all these have been discussed. An approximate solution has been obtained, for small Reynolds numbers, by expanding the unknowns as a power series in Reynolds number. The flow characteristics, like transverse velocity, radial velocity, pressure constant, shearing stresses, non-dimensional temperature distribution, and rates of heat transfer at both the disks, have been computed and sketched. The effects of porosity, relative angular velocities of the disk and Reynolds number etc, have been discussed. For example, it is shown that generally, the disks experience attraction but when the net effect of the porosity of walls is an axial inflow of liquid, they may suffer repulsion; the rate of heat transfer is smaller or greater at the hotter disk according as non-porous conditions exist or not. The rate of heat transfer increases with increase in relative angular velocity, decrease in mutual distance and decrease in temperature difference.

In the fifth chapter an exact numerical solution of the problem of 'Axially Symmetric Flow of Viscous Liquid Between Two Infinite Porous Disks' has been obtained by Runge-Kutta technique. The method of solution is valid for small as well as large values of Reynolds number, however, we have confined the solution upto Reynolds number equal to ten. The same problem has already been discussed in the fourth chapter, but therein the disks rotate slowly and we have obtained an approximate solution valid for small values of Reynolds number. A few distinguishing features are mentioned here. When both the disks are sucking the fluid and rotating in the opposite senses with equal angular velocities, the radial velocity has two maxima for Reynolds number equal to ten. When both disks are non-porous, transverse velocity and transverse shear stress are affected significantly at large Reynolds numbers but not at small Reynolds numbers.

The concluding chapter, deals with 'Flow Between an Unsteadily Rotating Infinite Disk and a Stationary Infinite Disk'. An exact numerical solution, of the resulting complicated non-linear differential equations, has been obtained by Runge-Kutta method. The governing differential equations emerge as two boundary value problems. The first problem governs the instantaneously steady state ( i.e., quasi-steady state ), and the second problem governs the first deviations of the actual instantaneous state from the quasi-steady state.

It has been found that these deviations in radial velocity, transverse velocity, axial velocity, shear stresses and pressure constant are of minor significance at small Reynold's numbers ($R$), but grow in stature with increasing value of $R$; so much so that for large values of $R$ these deviations become dominating. This corroborates the well known fact that unsteadiness of flows increases with increasing $R$. Criteria have also been developed for applying a quasi-steady state relationship to unsteady problems with desired accuracy.

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