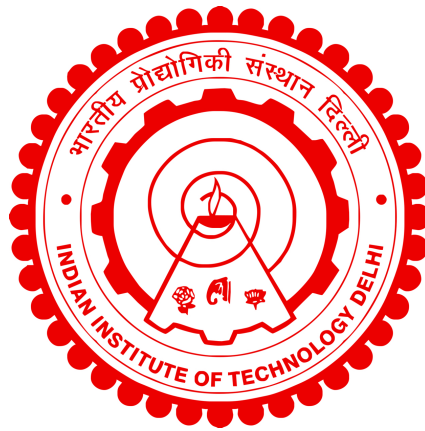


**STUDIES ON INTERRELATION AMONG
NONCLASSICAL FEATURES AND
APPLICATIONS TO QUANTUM
COMMUNICATION**

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DEPARTMENT OF PHYSICS

INDIAN INSTITUTE OF TECHNOLOGY DELHI

JUNE 2024

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Studies on interrelation among nonclassical features and applications to quantum communication

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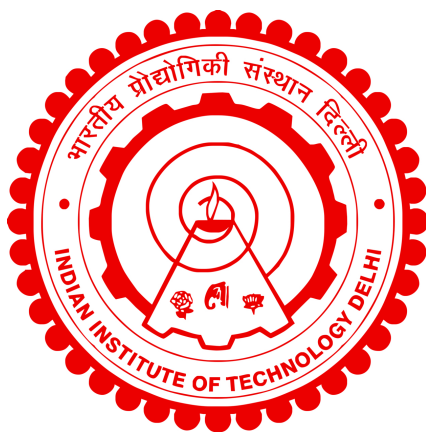
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Department of Physics

Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI

JUNE 2024

Dedicated to my parents

Certificate

This is to certify that the thesis titled “**Studies on interrelation among nonclassical features and applications to quantum communication**”, being submitted by **Sooryansh Asthana** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Physics, is a record of the original, bona fide research work carried out by him under my supervision and guidance. He has fulfilled the requirements for the submission of the thesis, which to the best of my knowledge has reached the required standard. The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.

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Date: 26 Jun 2024

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Sooryansh Asthana

Abstract

The area of quantum information processing has broadly two main research objectives: (i) unraveling novel quantum features for a better understanding of quantum foundations and, (ii) designing potential information processing applications with different quantum features as resources. Though the first objective dates back to the inception of quantum mechanics, it has received renewed research attention owing to the resource-theoretic importance of different quantum features. This, in turn, has led to a plethora of resource theories, with the resource theory of entanglement being the most celebrated example. We believe that it is timely to investigate whether criteria for different forms of quantumness (nonclassicality) may be arrived at through a unified framework. In this thesis, we accomplish this task by employing two frameworks—based on logical quantum systems and based on pseudo probability. In the framework based on logical quantum systems, we start with a simple observation: coherence in logical quantum systems is a manifestation of nonclassical correlations in the underlying physical qudits. For this reason, there is a reciprocity between the conditions for the former and the conditions for the latter. The reciprocity also permits a systematic construction of sufficiency conditions for nonclassical correlations. By employing a framework based on pseudoprobability, we show that all the criteria for multiparty nonlocality and entanglement can be looked upon as violations of classical probability rules. Importantly, these violations are not purely theoretical constructs, rather they can also be experimentally observed as anomalous weak values, thanks to rapid advancements in the area of weak measurements.

In the second part of the thesis, we turn our attention to the question: to what extent, can we relax the need for entanglement in quantum protocols and still get a quantum advantage, albeit in some restricted scenarios? This question stems from the fact that entanglement is a costly resource. We answer this question in two steps. In the first step, we leverage the counter-intuitive finding of [Bharath and Ravishankar. *Phys. Rev. A* **89.6** (2014): 062110] that an entangled quantum state can be simulated by a suitably higher-dimensional separable state. In fact, the two have the same information content. Based on this insight, we present protocols for quantum communication with $SU(2)$ invariant separable higher dimensional states. These protocols are designed for transferring information to remote locations. In the second step, we explore the

potential of reducing the requirement for quantum resources by expanding the domain of classicality to include classical waves. As a consequence, protocols requiring parallelism at the same spatial location can be simulated with classical waves as well. We take a step ahead and ask the question: can classically entangled beams be replaced by classically separable beams? We show that the answer to this question is in the affirmative and lies in laying down a classical optical version of the concept of simulability introduced in [Bharath and Ravishankar. *Phys. Rev. A* 89.6 (2014): 062110]. Notably, it is an area in which a quantum mechanical concept enriches the understanding in the classical realm.

सार

क्वांटम सूचना प्रसंस्करण के क्षेत्र में दो मुख्य अनुसंधान उद्देश्य हैं: (i) क्वांटम नींवों को बेहतर ढंग से समझने के लिए नए क्वांटम विशेषताओं का अनावरण करना, और (ii) विभिन्न क्वांटम विशेषताओं को संसाधनों के रूप में उपयोग करके संभावित सूचना प्रसंस्करण अनुप्रयोगों को डिजाइन करना। यद्यपि पहला उद्देश्य क्वांटम यांत्रिकी की शुरुआत से जुड़ा हुआ है, तथापि विभिन्न क्वांटम विशेषताओं के संसाधन-सैद्धांतिक महत्व के कारण इसे शोध में नवीन ध्यान प्राप्त हुआ है। इस कारण से कई संसाधन सिद्धांतों का उद्भव हुआ है, जिसमें व्यतिकरण (एंटेग्लमेंट) का संसाधन सिद्धांत सबसे प्रसिद्ध उदाहरण है। हम मानते हैं कि यह समय विभिन्न प्रकार की क्वांटमनैस (गैर-शास्त्रीयता) के लिए मानदंडों को एकीकृत ढांचे के माध्यम से जांच के लिए उपयुक्त है। इस शोधग्रंथ में, हम दो ढांचों का उपयोग करके इस कार्य को पूरा करते हैं— जो कि तर्कसंगत क्वांटम प्रणालियों (quantum logical systems) और छद्मप्रायिकता (pseudoprobability) पर आधारित हैं। तर्कसंगत क्वांटम प्रणालियों पर आधारित ढांचे में, हम एक सरल अवलोकन के साथ शुरू करते हैं: तर्कसंगत क्वांटम प्रणालियों में सामंजस्य (coherence) अंतर्निहित भौतिक क्यूडिट्स में गैर-शास्त्रीय सहसंबंधों (nonclassical correlations) का एक प्रकटीकरण है। इस कारण से, दोनों के लिए शर्तों के बीच एक पारस्परिकता है। यह पारस्परिकता गैर-शास्त्रीय सहसंबंधों के लिए पर्याप्तता शर्तों के व्यवस्थित निर्माण की अनुमति भी देती है। छद्मसंभावना पर आधारित ढांचे का उपयोग करके, हम दिखाते हैं कि सभी बहुपक्षीय गैर-स्थानीयता (nonlocality) और व्यतिकरण के मानदंडों को शास्त्रीय संभावना नियमों के उल्लंघन के रूप में देखा जा सकता है। महत्वपूर्ण बात यह है कि ये उल्लंघन केवल सैद्धांतिक निर्माण नहीं हैं, बल्कि वे कमजोर मापन (weak measurement) के क्षेत्र में तेजी से प्रगति के कारण विसंगत कमजोर मानों के रूप में प्रयोगात्मक रूप से भी देखे जा सकते हैं।

शोधग्रंथ के दूसरे भाग में, हम इस प्रश्न पर ध्यान केंद्रित करते हैं: किस सीमा तक, हम क्वांटम प्रोटोकॉल में व्यतिकरण की आवश्यकता को शिथिल कर सकते हैं और फिर भी कुछ प्रतिबंधित परिदृश्यों में क्वांटम लाभ प्राप्त कर सकते हैं? यह प्रश्न इस तथ्य से उत्पन्न होता है कि व्यतिकरण एक महंगा संसाधन है। हम इस प्रश्न का उत्तर दो चरणों में देते हैं। पहले चरण में, हम [Bharath and Ravishankar. Phys. Rev. A 89.6 (2014): 062110] के उस प्रतिकारक खोज का लाभ उठाते हैं कि एक व्यतिकृत

क्वांटम स्टेट को एक उपयुक्त उच्च-आयामी पृथक स्टेट द्वारा अनुकरण किया जा सकता है। वास्तव में, दोनों में एक ही सूचना सामग्री होती है। इस अंतर्दृष्टि के आधार पर, हम $SU(2)$ अपरिवर्तनीय उच्च आयामी पृथक अवस्थाओं के साथ क्वांटम संचार के लिए प्रोटोकॉल प्रस्तुत करते हैं। ये प्रोटोकॉल जानकारी को दूरस्थ स्थानों पर स्थानांतरित करने के लिए डिज़ाइन किए गए हैं। दूसरे चरण में, हम शास्त्रीयता के डोमेन को विस्तारित करके क्वांटम संसाधनों की आवश्यकता को कम करने की संभावना का पता लगाते हैं ताकि इसमें शास्त्रीय तरंगें शामिल हों। परिणामस्वरूप, समान स्थानिक स्थान पर समानांतरता की आवश्यकता वाले प्रोटोकॉल को शास्त्रीय तरंगों के साथ भी अनुकरण किया जा सकता है। हम एक कदम आगे बढ़कर पूछते हैं: क्या शास्त्रीय रूप से व्यतिकृत किरणों को शास्त्रीय रूप से पृथक किरणों से बदला जा सकता है? हम दिखाते हैं कि इस प्रश्न का उत्तर सकारात्मक है और यह [Bharath and Ravishankar. Phys. Rev. A 89.6 (2014): 062110] में प्रस्तुत सिमुलेबिलिटी की अवधारणा के शास्त्रीय ऑप्टिकल संस्करण को स्थापित करने में निहित है। विशेष रूप से, यह एक ऐसा क्षेत्र है जिसमें एक क्वांटम यांत्रिक अवधारणा शास्त्रीय क्षेत्र में समझ को समृद्ध करती है।

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