

OPTIMUM DESIGN OF MECHANICAL COMPONENTS UNDER STRESS USING  
FINITE ELEMENT ANALYSIS AND NONLINEAR PROGRAMMING

S.S. BHAVIKATTI

Applied Mechanics Department

Submitted

in fulfilment of the requirements of the  
degree of Doctor of Philosophy

to the  
Indian Institute of Technology, Delhi

September 1977

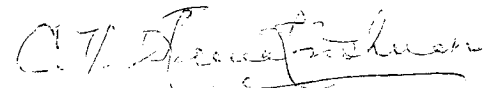
TO THE MEMORY

of

MY FATHER

## CERTIFICATE

This is to certify that the thesis entitled 'Optimum Design of Mechanical Components Under Stress Using Finite Element Analysis and Nonlinear Programming' being submitted by Shankar S. Bhavikatti to the Indian Institute of Technology, Delhi, India, for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of bonafide research work carried out by him under my supervision and guidance. The thesis work, in my **opinion** has reached the standard fullfilling the requirements for the Doctor of Philosophy Degree. The research report and results presented in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma.



(Dr. C.V. Ramakrishnan)  
Assistant Professor  
Department of Applied Mech.  
Indian Institute of Technology  
Hauz Khas, New Delhi-110029,  
India.

## ACKNOWLEDGEMENTS

It is with great pleasure that the author records his deep sense of gratitude to Dr. C.V. Ramakrishnan, Department of Applied Mechanics, Indian Institute of Technology, Delhi, who suggested the research problem and supervised the work reported in this thesis. His unceasing enthusiasm, fruitful discussions and constant encouragement inspired the author at all stages of this work.

The author is deeply grateful to Prof. M.K. Jain, Head of Computer Centre, Indian Institute of Technology, Delhi and his staff for the help rendered by them. He is particularly thankful to Messers J. Kurian, A.S. Dogra, A.K. Patra, Karamjit Singh, V.P. Singh and N.C. Saxena for their helpful cooperation.

The author is sincerely thankful to Prof. B.C. Nakra, former coordinator, Inter Disciplinary Program on Tribology, Machine Design and Dynamics and Prof. R.C. Malhotra, Head Design Subgroup, for providing necessary financial grant to use IBM 370 computer at Indian Institute of Technology, Madras.

The author is grateful to the authorities of Karnataka Regional Engineering College, Surathkal-574157, India, for sponsoring him for higher studies and the Ministry of Education, Government of India, for financial support under the Quality Improvement Programme.

Finally, the author thanks his near and dear ones, who very willingly, endured certain hardships during his preoccupation with this work.

## ABSTRACT

This study is concerned with the development of a suitable structural synthesis procedure for mechanical components under stress and its application to a few problems. The emphasis is mainly on the optimum design of shapes of the mechanical components, the objective being a better performance without violating constraints on stress, deflection and other design requirements. The geometries of such structures are very complicated and hence simple closed form solutions are not possible for stress analysis. Finite element method is considered to be ideally suited on account of its versatility in handling arbitrary boundary conditions, accuracy when used with superior elements and adaptability for different problems.

In the shape optimization of the continua the number of design variables is usually small but the number of constraints is large. The computing time required for each stress analysis is large. For such cases sequential linear programming is suited well. Three improvements are suggested in the conventional move limit method of sequential linear programming. The efficiency of the improved method is demonstrated in context of six test problems.

A general purpose optimization program SLPF is developed for the optimum shape design of mechanical components under stress. For stress analysis quadratic isoparametric elements are used and for optimization improved move limit

method of sequential linear programming is used. The stress derivatives are assembled by an efficient method. The program SLPF consists of a main program, 21 standard subroutines and six problem dependent subroutines. Five problem dependent subroutines are very small and need very little effort from the user.

This program has been used to investigate optimum shapes of the following five mechanical components (i) Fillets in flat and round tension bars (ii) Shoulder fillets in tension bars and T-heads; (iii) Cross section of rotating disk (iv) Flanged and flued expansion joints and (v) Transition profile of a spherical pressure vessel and nozzle junction. In last two problems multiple loadings are considered. In each problem the mathematical formulation, finite element idealization and progress of optimization are explained. For each problem, studies have been conducted by varying certain design parameters and the results are presented in detail. Due to limitations on computing time no attempt is made to prepare complete design charts. The capability of the program to handle various problems is clearly demonstrated. Useful design curves are given for each problem.

The efficiency of the improved method is compared with conventional method in the context of the structural problems. All the three suggested improvements are found to be very useful in structural problems and they have made the method more efficient and reliable.

## TABLE OF CONTENTS

ACKNOWLEDGEMENTS		i
ABSTRACT		iii
TABLE OF CONTENTS		v
LIST OF TABLES		xii
LIST OF FIGURES		xiv
LIST OF NOTATIONS		xviii
CHAPTER I	INTRODUCTION	1
	1.1 INTRODUCTORY REMARKS	1
	1.2 OPTIMUM DESIGN METHODS	2
	1.2.1 Experimental Design	2
	1.2.2 Conceptual Design	3
	1.2.3 Computer Aided Design	3
	1.2.4 Automated Optimum Design	3
	1.3 REVIEW OF TEXTS AND SURVEYS ON OPTIMUM STRUCTURAL DESIGN	3
	1.4 DETAILED REVIEW OF LITERATURE ON OPTIMUM STRUCTURAL DESIGN	
	1.4.1 Simultaneous Mode of Failure	6
	1.4.2 Criterion of Optimality	8
	1.4.3 Mathematical Programming	
	1.4.3.1 Linear Programming	10
	1.4.3.2 Nonlinear Programming	11
	1.5 THE OBJECT AND SCOPE OF PRESENT INVESTIGATION	17
	1.6 ORGANIZATION OF THE THESIS	18
CHAPTER II	PROBLEM FORMULATION	
	2.1 INTRODUCTION	21
	2.2 SPECIAL CHARACTERISTICS OF THE PROBLEM	21

<b>2.3 METHOD OF ANALYSIS</b>	
2.3.1 Choice of Method	23
2.3.2 General Theory	24
2.3.3 Choice and Use of the Elements	28
2.3.4 Evaluation of Boundary Stresses	31
<b>2.4 MATHEMATICAL FORMULATION</b>	
2.4.1 Design Variables and Geometric Description of the Problem	32
2.4.2 Objective Function	33
2.4.3 Constraints	
2.4.3.1 Behaviour Constraints	34
2.4.3.2 Side Constraints	34
2.4.4 Mathematical Representation of the Problem	35
<b>2.5 OPTIMIZATION TECHNIQUE</b>	
2.5.1 Choice of Optimization Technique	36
2.5.2 Sequential Linear Programming	37
<b>2.6 METHOD OF EVALUATING DESIGN DERIVATIVES</b>	
2.6.1 Derivatives of Jacobian and Jacobian Inverse Matrices	38
2.6.2 Derivative of [ B ] Matrix	39
2.6.3 Force Derivatives	40
2.6.4 Displacement Derivatives	40
2.6.5 Strain Derivatives	42
2.6.6 Stress Derivatives	43
2.6.7 Derivatives of Stress Constraints and Objective Function	43
<b>2.7 NEED FOR AUTOMATED MESH GENERATION</b>	45
<b>2.8 CLOSURE</b>	45
<b>FIGURES</b>	46

CHAPTER III	IMPROVED MOVE LIMIT METHOD OF SEQUENTIAL LINEAR PROGRAMMING	
3.1	INTRODUCTION	50
3.2	SEQUENTIAL LINEAR PROGRAMMING	51
3.2.1	Cutting Plane Method	53
3.2.2	Move Limit Method	53
3.3	SUGGESTED IMPROVEMENTS	55
3.3.1	Quadratic Interpolation Along a Line	55
3.3.2	Steering of the Design Vector into Feasible Domain	57
3.3.2.1	When Feasible Region is not Reached in one Attempt	59
3.3.2.2	When Limits on the Variables Are Violated	60
3.3.3	Checking the Usability of the New Direction	60
3.4	COMPUTER PROGRAM AND FLOW CHART	61
3.5	EXAMPLES	63
3.6	COMPARISON WITH CONVENTIONAL MOVE LIMIT METHOD	67
3.7	CLOSURE	69
	TABLES	70
	FIGURES	73
CHAPTER IV	THE PROGRAM SLPF	
4.1	INTRODUCTION	83
4.2	GENERAL ORGANIZATION OF THE PROBLEM	84
4.2.1	Data Processor	85
4.2.2	Optimizer	85
4.2.3	Analyser	86

4.3	BRIEF DESCRIPTION OF THE PROBLEM	87
4.3.1	Data Processor	87
4.3.2	Optimizer	88
4.3.3	Analysar	88
4.3.3.1	Calculation of Load Vector and Element Stiffness Matrix	89
4.3.3.2	Equation Solver	89
4.3.3.3	Calculation of Stresses	90
4.3.3.4	Evaluation of Design Derivatives	90
4.3.4	Imposition of Side Constraints	91
4.4	DIFFICULTIES EXPERIENCED	91
4.5	SALIENT FEATURES OF THE PROGRAM	92
4.6	DEFICIENCIES OF THE PROGRAM AND SUGGESTIONS FOR FURTHER IMPROVEMENTS	93
4.7	EXPERIENCE IN USING THE PROGRAM SLPF	94
4.8	CLOSURE	95
	FIGURES	96
CHAPTER V	OPTIMUM SHAPE DESIGN OF FILLETS IN FLAT AND ROUND TENSION BARS	
5.1	INTRODUCTION	99
5.2	GEOMETRIC DESCRIPTION OF FILLET PROFILE	99
5.3	FINITE ELEMENT ANALYSIS AND AUTOMATED MESH GENERATION	101
5.4	CHOICE OF OBJECTIVE FUNCTIONS AND CONSTRAINTS	102
5.5	NUMERICAL EXPERIMENTATION WITH DIFFERENT OBJECTIVE FUNCTIONS	103
5.6	DESIGN CHARTS	106
5.6.1	Design Charts for Flat Tension Bars	106
5.6.2	Design Charts for Round Tension Bars.	107

	5.7 DISCUSSION	107
	5.8 DETAILS OF COMPUTING TIME	108
	5.9 CLOSURE	109
	TABLES	110
	FIGURES	117
CHAPTER VI	OPTIMUM SHAPE DESIGN OF SHOULDER FILLETS AND T-HEADS	
	6.1 INTRODUCTION	127
	6.2 GEOMETRIC DESCRIPTION OF CURVED PROFILE	128
	6.3 FINITE ELEMENT ANALYSIS	131
	6.4 CHOICE OF OBJECTIVE FUNCTIONS AND CONSTRAINTS	132
	6.5 PROGRESS OF OPTIMIZATION	132
	6.6 RESULTS FOR SHOULDER FILLET	133
	6.7 RESULTS FOR T-HEAD	134
	6.8 DETAILS OF COMPUTING TIME	134
	6.9 CLOSURE	135
	TABLES	136
	FIGURES	145
CHAPTER VII	OPTIMUM SHAPE DESIGN OF ROTATING DISKS	
	7.1 INTRODUCTION	156
	7.2 GEOMETRIC DESCRIPTION OF CROSS SECTION OF ROTATING DISKS	157
	7.3 FINITE ELEMENT ANALYSIS	158
	7.4 CHOICE OF OBJECTIVE FUNCTION AND CONSTRAINTS	159
	7.5 CHOICE OF NON DIMENSIONAL VARIABLES	162
	7.6 PROGRESS OF OPTIMIZATION	163
	7.7 RESULTS AND PARAMETRIC STUDIES	164
	7.8 DETAILS OF COMPUTING TIME	165
	7.9 CLOSURE	165
	TABLES	166
	FIGURES	168

CHAPTER VIII	OPTIMUM DESIGN OF FLANGED AND FLUED EXPANSION JOINTS	176
	8.1 INTRODUCTION	176
	8.2 A TYPICAL FLANGED AND FLUED EXPANSION JOINT	177
	8.3 FINITE ELEMENT ANALYSIS	178
	8.4 DESIGN CRITERIA	179
	8.5 CHOICE OF OBJECTIVE FUNCTION AND CONSTRAINTS	180
	8.6 CHOICE OF NON DIMENSIONAL VARIABLES	181
	8.7 PROGRESS OF OPTIMIZATION	182
	8.8 RESULTS AND PARAMETRIC STUDIES	184
	8.9 DETAILS OF COMPUTING TIME	185
	8.10 CLOSURE	185
	TABLES	186
	FIGURES	188
CHAPTER IX	OPTIMUM SHAPE DESIGN OF PRESSURE VESSEL AND NOZZLE JUNCTION	
	9.1 INTRODUCTION	196
	9.2 GEOMETRIC DESCRIPTION OF JUNCTION	196
	9.3 CHOICE OF OBJECTIVE FUNCTION AND CONSTRAINTS	200
	9.4 FINITE ELEMENT ANALYSIS AND OPTIMI- ZATION PROCEDURE	202
	9.5 PROGRESS OF OPTIMIZATION	203
	9.6 RESULTS	204
	9.7 EFFECT OF AXIAL THRUST ON OPTIMUM SHAPE	205
	9.8 DETAILS OF COMPUTING TIME	207
	9.9 CLOSURE	207
	TABLES	208
	FIGURES	210

CHAPTER X	COMPUTATIONAL EFFICIENCY OF IMPROVING MOVE LIMIT METHOD OF SEQUENTIAL LINEAR PROGRAMMING	
	10.1 INTRODUCTION	222
	10.2 STRUCTURAL PROBLEMS CONSIDERED	222
	10.3 QUADRATIC INTERPOLATION	223
	10.4 STEERING INFEASIBLE VECTOR TO FEASIBLE REGION	224
	10.5 NEED FOR CHECKING THE USABILITY OF THE NEW DIRECTION	225
	10.6 CLOSURE	225
	TABLES	226
CHAPTER XI	SUMMARY AND CONCLUSIONS	228
REFERENCES		232
BIODATA		258