

A THESIS ON
VIBRATION OF ROTATING BLADES WITH SMALL
ASPECT RATIO

by

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submitted

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(i)

CERTIFICATE

This is to certify that the thesis entitled
'VIBRATION OF ROTATING BLADES WITH SMALL ASPECT RATIO'
by Kshitij Gupta has been prepared under my supervision
in conformity with the rules and regulation of the Indian
Institute of Technology, Delhi. I further certify that
the thesis has attained a standard required for a Ph.D.
degree of the Institute. The results contained in this
thesis have not been submitted, in part or full, to any
other university for any degree or diploma.

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VIBRATION OF ROTATING BLADES WITH SMALL ASPECT RATIO

A B S T R A C T

The natural frequencies of small aspect ratio pre-twisted plates have been determined by using a thin shell theory. The natural frequencies have been obtained by three separate methods viz., Levy, Inextensional and Ritz type.

In Levy's analysis, rigid body displacement for bending modes, rigid body rotation for torsional modes and free-free beam functions for plate modes have been used. In the second analysis, primarily for fundamental bending mode, inextensional theory of vibration of shells has been used. Third analysis uses the classical bending theory of thin shells with the help of Ritz method. The results of the third analysis are shown to be most accurate and reliable.

The natural frequencies have also been determined experimentally for stationary blades. The experimental results match well with the trends of the results presented by the theoretical analysis. It is shown that twist has a strong effect on coupling between different natural modes. The technique of Holographic Interferometry has been used to identify the modes of vibration.

On the basis of the theoretical and experimental results presented in present thesis, a controversy regarding the effect of pre-twist on bending natural frequencies of small aspect ratio pre-twisted blades is resolved.

Two formulations for studying the effect of centrifugal forces on natural frequencies of small aspect ratio blades are presented, which are shown to be same for small amplitude vibrations. The results from the present analysis are compared with the theoretical and experimental results of rotating pre-twisted plates available in literature. Southwell coefficient for different cases have been obtained and are found to decrease with pre-twist.

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