

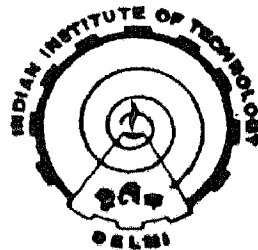
LINEAR AND PARAMETRIC INSTABILITIES OF WHISTLER WAVES IN A PLASMA

By

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Thesis submitted in fulfilment
of the requirements for
the degree of

DOCTOR OF PHILOSOPHY



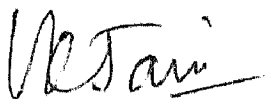
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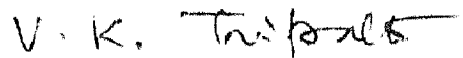
This is to certify that the thesis entitled "Linear and Parametric Instabilities of Whistlers In a Plasma" being submitted by Shri Indranil Talukdar to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy, is a record of bonafide research work carried out by him.

Indranil Talukdar has worked under our guidance and supervision and has fulfilled the requirements for the submission of this thesis, which to our knowledge, has reached the requisite standard.

The results contained in this thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.



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PREFACE

The excitation of electromagnetic instabilities in plasmas has been a subject of great interest in last few decades. Of considerable importance has been the propagation and excitation of Whistler waves, which are right hand circularly polarized electromagnetic wave having phase velocity smaller than the velocity of light in vacuum and are guided by the magnetic field. In solid state plasma these waves, known as helicons, are useful for diagnostics. In space they are useful for probing remote regions of ionosphere and magnetosphere. The Whistler method of measuring magnetospheric electron density has been described by a number of authors¹⁻³. A technical report by Park⁴ provides a valuable basis for numerical and graphical analysis work with Whistlers. Many authors investigated Whistlers emitted by ground transmitter and propagating along the field lines in the magnetosphere^{1,5,6}. The fine structure of Whistler spectra generated in unstable regions of magnetosphere have also been studied⁷. Similar phenomena for ion Whistler have been studied by Keney and Knaflich⁸.

Whistler propagation through magnetosphere is generally classified in two categories viz ducted propagation and non-ducted propagation. Ducted propagation is that in which the Whistler waves travel from one hemisphere to the other

in field-aligned columns of enhanced or depressed electron density whereas non-ducted propagation occurs in smoothly varying ionization⁹. The duct propagation of Whistler through the earth magnetosphere has generated considerable interest in recent years¹⁰⁻¹². A large number of experiments on naturally occurring Whistler and artificially launched VLF waves have observed this phenomena¹³⁻¹⁵. Patel and Tripathi¹⁶ have studied the stability of a Whistler duct in magnetosphere. The duct could be preformed or created by Whistler itself. In preformed duct several people have examined propagation and excitation of Whistler by beams, currents and temperature anisotropy¹⁷⁻¹⁸. In self made duct non-linear phenomena like self-focusing, filamentation and parametric instabilities are important. Stenzel¹⁹ has carried out extensive experiments on the self-focusing of Whistlers in large laboratory plasma. He observed self focusing of Whistler mode in the frequency interval $0.2 < \omega/\omega_c < 0.9$ where ω and ω_c are the wave and electron cyclotron frequencies respectively. A high amplitude Whistler with non-uniform intensity distribution along its wavefront exerts a ponderomotive force and a temperature gradient force on the electrons causing density redistribution such that the plasma density increases (decreases) in regions of higher intensity for $\omega < \omega_c$ ($\omega > \omega_c$). The effective permittivity of the plasma is accordingly modified, leading to the process of non-linear refraction.

The electromagnetic energy is attracted towards the regions of higher intensity causing self-focusing. The tendency of self convergence is countered by the diffraction effects. A balance between the two processes leads to self duct propagation. The linear trapping of Whistler waves in field aligned density crests has been investigated experimentally and theoretically, below $\omega = \omega_c/2$ by Sugai et al²⁰. At high powers they observed undamped non-diverging Whistler wave together with a field aligned trough and electron heating. However, the density trough is found not only in the wave propagation regime ($\omega/\omega_c < 1$) but also in wave evanescence regime ($\omega/\omega_c > 1$).

Active VLF wave injection experiments have contributed greatly to the understanding of VLF wave-plasma interactions in the magnetosphere. In many cases, it is observed that the Whistlers interact with the energetic electrons encountered in the duct, resulting in pitch angle scattering of electrons by the wave and subsequent precipitation of energetic electrons in the atmosphere²¹. This causes ionization and conductivity enhancement, heating and the emission of X-rays and optical frequencies.

The study of antennas in plasma has also been a topic of great interest. Particular attention has been given to VLF and ELF radiation due to its importance in space craft applications and submarine communication. Many authors²²⁻²⁸

have worked on the theory of antenna radiation problem. However, lot of more needs to be done in this field before it could be exploited for commercial applications.

Another area in which Whistler have emerged as very potential candidate is the RF heating and current drive in tokamak. The steady state operation of a tokamak requires a non-inductive toroidal current ($\approx 1\text{MA}$) which should be generated in an efficient way²⁹. The generation of the current with r.f. wave has been shown both feasible and efficient. In low density tokamak operation lower hybrid waves are quite efficient giving 1 Ampere current per watt of r.f. power³⁰. However, there exists a strong density threshold above which current cannot be driven³¹. Whistler waves, however, can operate upto much higher densities, hence, are a potential alternative to lower hybrid wave for current drive³²⁻³⁴. A detailed theory of current drive with Whistler waves has been studied by deAssis and Bunard-Neto³⁵.

There is another dimension to Whistler wave research. The emission of electromagnetic radiation is a valuable tool for understanding plasma conditions³⁶, to provide a seed of growth of low frequency turbulence³⁷⁻³⁹ and perhaps to establish communication link as may be the case in the ionosphere. In tokamak plasma, where the magnetic field \vec{B} is

a function of radius, emissions perpendicular to \vec{B} at the electron cyclotron frequency $\omega_c = e\vec{B}/m$ are used to measure electron temperature T_e as a function of radius⁴⁰. However, in a tandem mirror with thermal barriers⁴¹ T_e varies axially, i.e. along \vec{B} . No practical diagnostics now exists to measure this variation and time dependence. Ellis et al.⁴² have showed that electron cyclotron emission parallel to \vec{B} field in the Whistler mode is well suited because a) in an axially varying \vec{B} the condition $\omega = \omega_c$ means that different frequencies corresponds to different axial locations; b) it propagates at high densities ($\omega_p > \omega_c$), where $\omega_p = (4\pi n e^2/m)^{1/2}$ and n is the plasma density c) emission reaches black-body levels for a wide range of plasma parameters and depends only on the electron temperature and not on other parameters such as plasma density.

Recently Whistlers have been proposed as serious candidates for free electron laser Wigglers. Whistlers with frequencies close to electron cyclotron frequency can produce much higher frequency FEL radiation at not too high beam voltage. Furthermore, the FEL instability could have an explosive growth giving a much higher efficiency than a conventional FEL⁴³.

Excitation of coherent electromagnetic wave by modest

energy particle beam has been an area of considerable interest. One such scheme is a travelling wave tube. Another even more attractive scheme is a magnetized beam plasma system which supports slowly propagating Whistlers that can be driven (excited) by medium energy electron beams⁴⁴⁻⁴⁷. It is important in these devices to include non-local effects. Newman et al.⁴⁵ provided a fuller treatment of evaluation of parallel propagating low frequency electromagnetic wave by a thermally anisotropic beam in magnetised plasma. The excitation of Whistler in smaller laboratory devices has been experimentally investigated by Stenzel⁴⁴. Jain et al.⁴⁸ theoretically studied the excitation Whistler instability by a hellical electron beam.

The Whistlers exhibit non-linear effects at moderate power densities. The phenomena of self-focusing, filamentation, oscillating two stream instabilities and three wave parametric processes in unbounded plasmas have been studied in considerably detail over the years⁴⁹⁻⁵⁴. Recently a new four wave process viz., electromagnetic filamentation instability of a Whistler wave has been studied by Sharma and Patel⁵³. Krapman et al.⁵⁵ have given an elegant theory of non-linear effects of a quasimonochromatic Whistler wave packet in homogeneous plasma. In many experiments it is indicated that lower hybrid waves exist in the regions of ducted Whistler propagation through the

magnetosphere. Lee and Kuo⁵⁶ and Tripathi et al.⁵⁷ attributed it to short wavelength two stream instability (OTSI) in which a Whistler excites a purely growing short wavelength mode and two lower hybrid side bands. Patel and Tripathi⁵⁸ have investigated the process of self focused duct propagation of the Whistlers in the magnetosphere. Tripathi⁵⁹ has also developed a theory of parametric instability of fast wave (Whistler) in a tokamak.

With this brief review of the work done on Whistlers in the past, it would be in order to elaborate on some typical features of this wave. It is the X-mode (right hand circularly polarized) of electromagnetic wave propagation with frequency ω in the range $\omega_c > \omega > \omega_{ci}$, where ω_{ci} is the ion-cyclotron frequency. For propagation along the d.c. magnetic field it is a purely transverse wave $\vec{k} \cdot \vec{E} = 0$, and the electric field can be expressed as

$$\vec{E} = \vec{A} e^{-i(\omega t - kz)},$$

$$E_x = -iE_y,$$

$$k = \omega/c \left[1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \right]^{1/2}$$

For $\omega \ll \omega_c, \omega_p$

$$k \approx -\frac{\omega_p}{c} (\omega/\omega_c)^{1/2}$$

It is clear from the dispersion relation that as ω approaches ω_c , k goes to ∞ and v_{ph} tends to zero. The ω - k diagram of a Whistler wave is shown in Fig. 1.

At oblique angles, i.e. when the direction of propagation is at an angle θ with the static magnetic field, the Whistler has a longitudinal component of electric field, $\vec{k} \cdot \vec{E} \neq 0$ and

$$k = \frac{\omega}{c} \left\{ \frac{\omega^2}{\omega(\omega_c |\cos\theta| - \omega)} \frac{P}{\omega^2} \right\}^{1/2}$$

$$\approx \frac{\omega}{c} \left\{ \frac{\omega}{\omega(\omega_c |\cos\theta|)} \frac{P}{\omega^2} \right\}^{1/2}$$

Last equality is valid when $\omega \ll \omega_c$. The phase and group velocities can be written as

$$v_{ph} = \frac{\omega}{k} = \frac{c}{\omega} \frac{\omega^2}{\omega(\omega_c |\cos\theta| - \omega)} \frac{P}{\omega^2}$$

$$v_g = \frac{2\omega}{k} = \frac{2c}{\omega} \frac{\omega^2}{\omega(\omega_c |\cos\theta| - \omega)} \frac{P}{\omega^2}$$

The phase velocity of Whistler is always less than the velocity of light. At $\omega \ll \omega_c$ the group velocity is twice the phase velocity. It is important to note that the direction of group velocity lies closer, to the magnetic field than the direction of phase velocity. If we denote the

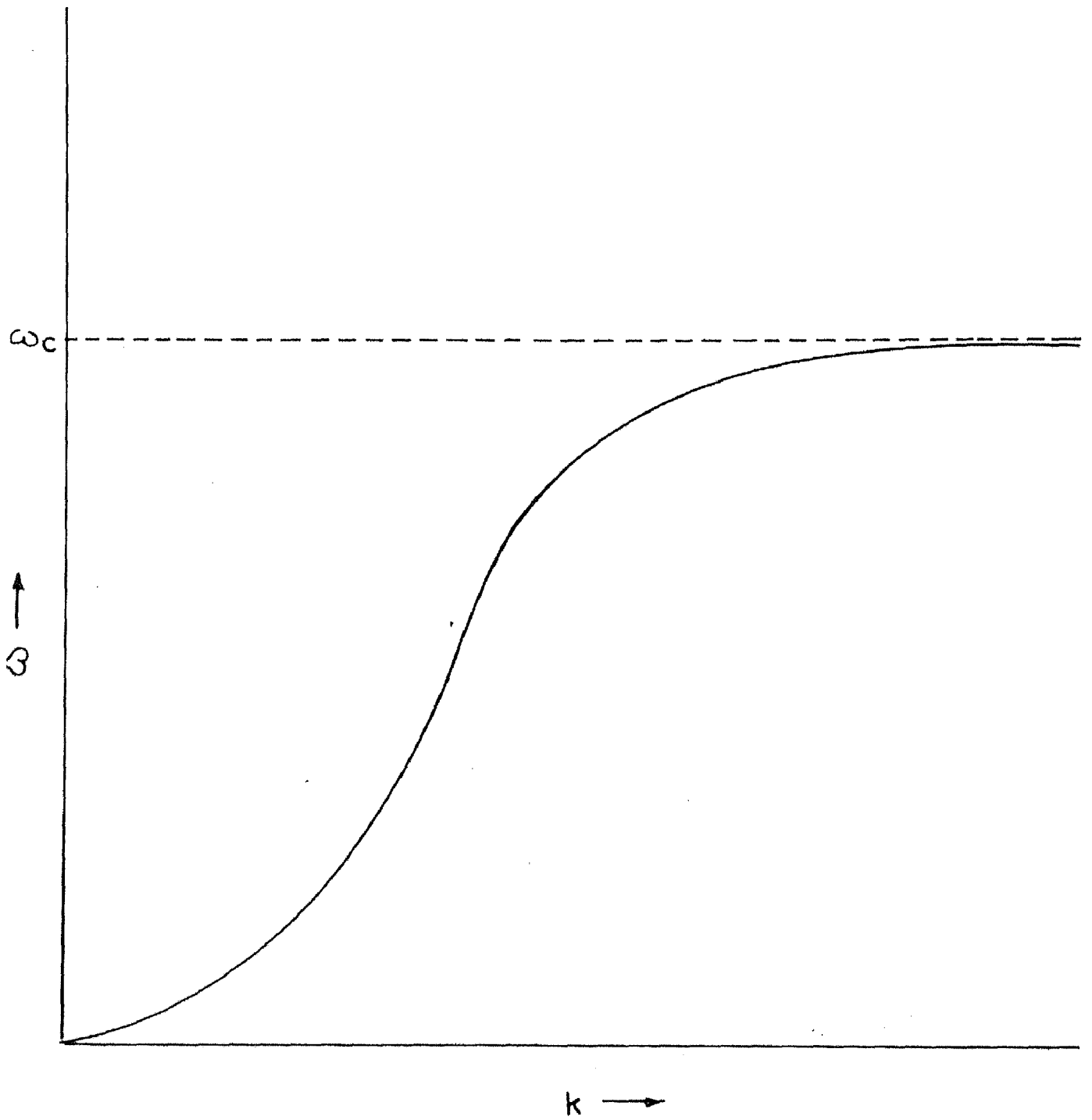


Fig.1. Dispersion curve (ω - k) of whistler wave for $\omega \ll \omega_c$

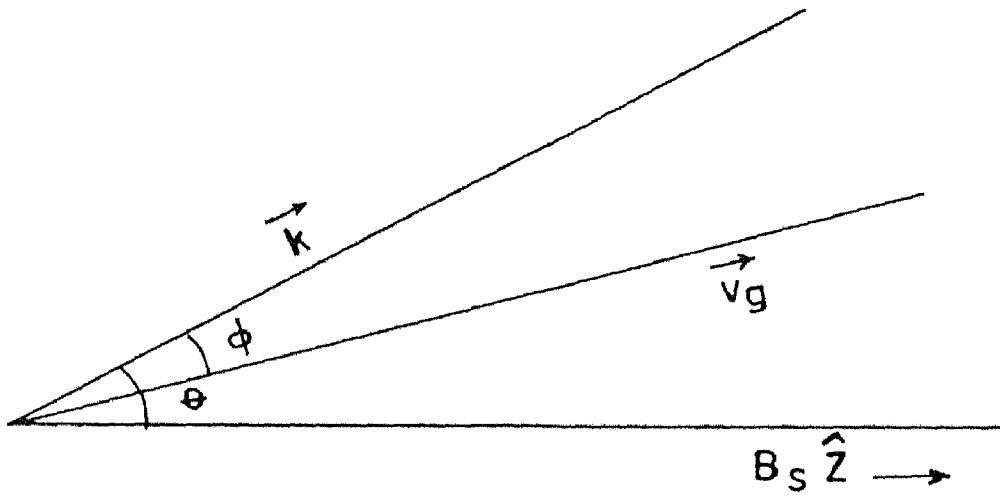


Fig.2. Oblique propagation of whistler wave

angle between the phase and group velocities by ϕ (c.f. Fig.2), we have $\tan(\theta - \phi) = 1/2 \tan\theta$.

Most of the work done on Whistler in the past is valid in an unbounded plasma where non-local effects are not important. This is a good approximation to large devices and space plasmas, however, it is not applicable to smaller devices like a beam plasma system or a mirror machine. In these devices the mode structures of interacting waves play an important role in wave particle interaction and must be properly accounted for. In the present thesis we have studied i) the excitation of Whistlers by electron beams and ii) Parametric instabilities of Whistler, in a bounded plasma employing analytical and numerical techniques. The thesis is organised as follows. It comprises of five chapters.

In Chapter-I we present a non-local theory of Whistler and quasi-static modes excitation by a thin annular electron beam in a cylindrical plasma. The plasma is immersed in a d.c. magnetic field $B_s \hat{z}$ and of uniform density n_0 , radius R . At $r=R$ there is a metallic boundary. The annular electron beam of radius r_0 and density $n_b = n_{b0} (r-r_0)$ is propagating along the z -axis with velocity $v_b \hat{z}$. The number of beam electrons per unit axial length is much smaller than that of the plasma electrons i.e. $n_b r_0 \ll n_0 R^2/2$. This configuration is very easily realized in laboratory⁴⁴. The first order perturbation technique has been used to find out the growth

rate. The instability possesses maximum growth when the wave function peaks at the radial position of the beam. It is found for a given beam radius the growth rate is too sensitive to the beam velocity. The variation of growth rate with beam radius is found to be large. The optimum beam radius is one for which the location of the mode amplitude maximum and beam's radial position coincide.

In Chapter II, the problem of excitation of Whistler in a plasma loaded sheath-helix by an electron beam has been studied. A metallic cylindrical waveguide of radius R_c , containing a helical wire of radius R_h inside it, is considered. The plasma is located between $0 < r < R_p$. It is of uniform density and is cold. An axial magnetic field $\vec{B}_s \parallel \hat{z}$ is applied on the plasma. A uniform electron beam of radius R_b is propagating inside the plasma along the z-axis with velocity $v_b \hat{z}$. The number of beam electrons per unit axial length is much smaller than that of the plasma electrons. The helix slows down the Whistler mode and relaxes the requirement on the beam energy to excite the Whistler mode. The interaction occurs via slow-cyclotron resonance. The slowing down effect of the helix increases with the decrease of the separation between consecutive turns of the helix. The plasma radius is chosen to be smaller than the helix radius, while the gap between them is smaller than a free space wavelength. The growth rate of the

instability falls off with the increase in the frequency of the Whistler.

Chapter III deals with the non-local theory of Whistler instability driven by an electron beam in a two dimensional magnetospheric duct. The duct has a parabolic density profile in which the trapped Whistler has Hermit wave functions. In the presence of the beam the mode structure is not too much altered but the eigen values are. Hence a perturbation theory is employed to obtain the change in eigen values and the growth rate. Two profiles of transverse beam density variation are considered viz. a flat density profile and an exponential density profile. It is seen that a ducted Whistler by virtue of its localization in transverse direction possesses an effective k_{\perp} and can resonantly interact with an axial beam. The interaction occurs via cerenkov and slow-cyclotron resonances. For beam density profile peaking at the center the largest growth rate occurs for the mode that has field maximum at the axis, i.e., only fundamental mode is excited.

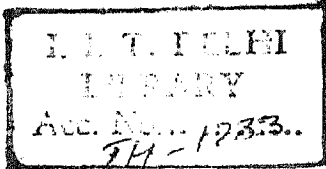
In Chapter IV the possibility of existence and excitation of low frequency electron acoustic wave in a electron beam plasma-system has been studied. The plasma is unmagnetised and of infinite extent. The beam is also uniform. A new spectrum of electromagnetic mode of low

frequency is found. Two cases are considered: 1) cold plasma hot beam and ii) hot plasma cold beam. The spectra obtained in the two cases differ considerably.

In Chapter V we develop a non-local theory of parametric instability of Whistler in a plasma cylinder. Two specific channels of decay are considered, viz., the resonant decay into ion-cyclotron modes and non-resonant decay into ion-cyclotron quasi-modes. In both cases side band is a lower-hybrid mode. The finite radial extent of the plasma introduces non-local effects and suppresses the growth rate significantly. However, it eliminates side ways convective losses and reduces the threshold for the onset of parametric instability. For a typical Whistler wave of 7.5×10^9 rad/sec, 50 watt, plasma densities $\approx 10^{12}$ cm⁻³, magnetic field ≈ 10 KG and plasma radius ≈ 3 cm, the growth rate turns out to be $\approx 10^{-4} \omega_c$. The resonant decay into ion-cyclotron mode is relatively less important.

The work mentioned in the thesis has resulted in the following publications:

1. Excitation of Whistlers and quasi-static modes by an annular electron beam in a plasma cylinder:
I. Talukdar and V.K. Jain, J. Plasma Phys. 31, 407 (1988).
2. Excitation of Whistler modes in a sheath-helix loaded waveguide:
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3. Whistler instability in a magnetospheric duct:
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4. The possibility of low frequency electron waves in a beam plasma system:
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G. Mishra, I. Talukdar, Vijayshree and V.K. Tripathi, IEEE Trans. on Plasma Sci. (To be appear in December 1989). (Not included in the thesis).
7. Modulation instability of Trivelpiece-Gould mode in a Beam plasma system:
G. Praburam, G. Mishra, I. Talukdar, A.K. Sharma, V.K. Jain, Communicated to J. Plasma Phys. (Not included in the thesis).

CONTENTS

Chapter		Page
	PREFACE	1-19
I	EXCITATION OF WHISTLER AND QUASI-STATIC MODES BY AN ANNULAR ELECTRON BEAM IN A PLASMA CYLINDER	20-33
1.1	Introduction	20
1.2	Mode Structure	21
1.3	Growth Rate	25
1.4	Discussion	29
	References	33
II	EXCITATION OF WHISTLER MODES IN A SHEATH-HELIX LOADED WAVE GUIDE	34-53
2.1	Introduction	34
2.2	Dispersion Relation	35
2.3	Growth Rate	44
2.4	Discussion	45
	References	51
	APPENDIX	52
III	WHISTLER INSTABILITY IN A MAGNETOSPHERIC DUCT	54-84
3.1	Introduction	54
3.2	Local Theory	56

3.3	Non-local Theory	61
3.4	Discussion	68
	References	70
	APPENDIX	71-84
	KINETIC THEORY OF WHISTLER	
	INSTABILITY	
	A.1 Modified Distribution Function	72
	A.2 Beam Current Density	75
	A.3 Growth Rate	78
IV	THE POSSIBILITY OF EXCITATION OF LOW	
	FREQUENCY ELECTRON WAVE IN A BEAM PLASMA	
	SYSTEM	85-95
4.1	Introduction	85
4.2	Instability Analysis	86
4.3	Discussion	94
	References	95
V	QUASIMODE AND RESONANT DECAY OF WHISTLER	
	IN A PLASMA CYLINDER	96-116
5.1	Introduction	96
5.2	Instability Analysis	98
5.3	Discussion	111
	References	116