

**STABILITY AND DYNAMICS OF COMPOSITE PLATES  
AND CYLINDRICAL PANELS**

by

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## **CERTIFICATE**

This is to certify that the thesis entitled “*Stability And Dynamics of Composite Plates and Cylindrical Panels*”, that is being submitted by **Shri Rupesh Daripa** to the Indian Institute of Technology Delhi for the award of the degree of *Doctor of Philosophy* is a record of original research work carried out by him under my guidance and supervision.

Shri Daripa has fulfilled all the prescribed requirements and the thesis is, in my opinion, worthy of consideration for the degree of *Doctor of Philosophy* in accordance with the regulations of the Institute.

The contents of this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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## Abstract

Plates and shells, made of isotropic or laminated composite materials are used as major load carrying components of aerospace, automobile, submarine, civil, offshore and other important engineering structures. Hence, the stability and dynamic characteristics of such thin-walled structures is an important problem to be understood well for their effective and lightweight design.

In this thesis, an attempt is made to investigate some of the characteristic features of the nonlinear stability and dynamics of plates and cylindrical shell panels using the shear deformable finite element approach. For this purpose, one high precision shear flexible plate bending element with fourteen degrees of freedom per node is developed. The nonlinear stiffness matrix is formulated based on von Kármán's strain-displacement assumptions to obtain the stiffness interaction (coupling) between the in-plane and bending degrees of freedom. Then, the plate bending element is modified using Marguerre's strain-displacement assumptions and a shallow shell element is obtained. Further, the sixteen-noded MITC (mixed interpolation of tensorial components) shell element based on assumed strain fields (initially proposed by *Bucalem and Bathe* 1993) is extended here for the nonlinear vibration analysis of composite cylindrical panels.

At the beginning, the critical buckling load and the subsequent postbuckling behavior (*i.e.*, the lateral displacement and end-shortening) of isotropic and composite plates under static in-plane load are studied in this thesis. Numerical investigation is focused to investigate the influences of (a) different in-plane boundary conditions, (b) per-buckling corner stresses in the skew plate, (c) position and distribution of

localized in-plane edge load for the rectangular plate, (d) lamination parameter and (e) sinusoidal imperfection.

Nonlinear dynamics of plates and shells is a complex phenomenon involving both flexural vibration and in-plane strain. Attempt is made in this thesis to obtain the steady-state un-damped vibration characteristics of composite plates and shell panels. Approximate time functions (corresponding to a steady-state periodic oscillation) for the displacement components are assumed and a nonlinear matrix amplitude equation is obtained by employing Galerkin's method. The coupled nonlinear matrix amplitude equation (in-plane motion is coupled with flexural motion) is solved to estimate

- (a) nonlinear free flexural vibration frequencies of isotropic and composite plates with different in-plane boundary conditions,
- (b) flexural vibration amplitudes of such plates under transverse harmonic pressure,
- (c) flexural vibration amplitudes of such plates under periodic in-plane load,
- (d) vibration amplitude *versus* disturbing energy relationship for the case of free flexural vibration of cylindrical shell panels,
- (e) Frequency-amplitude relationship of cylindrical shell panels under radial harmonic pressure.

Finally, the time history analysis is carried out in each case to understand the steady-state or unsteady nature of the flexural vibration of plates and shell panels under different loading and boundary condition.

# 1 TABLE OF CONTENTS

<b>Title</b> .....	<b>i</b>
<b>Certificate</b> .....	<b>ii</b>
<b>Acknowledgements</b> .....	<b>iii</b>
<b>Abstract</b> .....	<b>iv</b>
<b>Table of contents</b> .....	<b>vi</b>
<b>List of figures</b> .....	<b>x</b>
<b>List of tables</b> .....	<b>xvi</b>

## CHAPTER: 1

<b>INTRODUCTION</b> .....	<b>1</b>
1.1 Introduction to the stability and dynamics of plates and shells .....	1
1.2 Engineering applications of composite materials .....	2
1.3 Present study - its importance in research .....	3
1.4 Remarks .....	4

## CHAPTER: 2

<b>LITERATURE REVIEW</b> .....	<b>5</b>
2.1 Introduction .....	5
2.2 Stability analysis of plates .....	5
2.3 Nonlinear vibration of plates .....	11
2.4 Nonlinear dynamic stability .....	16
2.5 Nonlinear vibration of cylindrical panels .....	17

## CHAPTER: 3

<b>SCOPE OF THE PRESENT INVESTIGATION</b> .....	<b>23</b>
3.1 Introduction .....	23
3.2 Solution procedure .....	23
3.3 Problems considered in the present study .....	24

3.4 Presentation and discussion of results .....	25
 <b>CHAPTER: 4</b>	
<b>SOLUTION METHODOLOGY .....</b>	<b>27</b>
4.1 Introduction .....	27
4.2 Finite element formulation .....	27
4.2.1 Plate bending finite element .....	27
4.2.2 Shallow Shell Element .....	31
4.2.3 Sixteen noded MITC shell element .....	32
4.3 Stability analysis of composite plates .....	35
4.4 Large amplitude flexural vibration of symmetrically laminated plates ...	36
4.5 Nonlinear dynamic stability analysis of composite plates .....	39
4.6 Large amplitude flexural vibration of laminated cylindrical panels .....	42
 <b>CHAPTER: 5</b>	
<b>STABILITY ANALYSIS OF COMPOSITE PLATES SUBJECTED TO IN-PLANE LOAD .....</b>	<b>45</b>
5.1 Introduction .....	45
5.2 Validation and convergence study .....	45
5.3 Stability analysis of composite skew plates .....	52
5.4 Stability analysis of rectangular composite plates under localized in-plane Load .....	62
5.5 Summary .....	67

## **CHAPTER: 6**

<b>NONLINEAR VIBRATION OF PLATES UNDER TRANVERSE LOAD</b>	<b>69</b>
6.1 Introduction	69
6.2 Validation and convergence study	69
6.3 Nonlinear free and forced vibration of rectangular plates	77
6.4 Nonlinear vibration of initially stressed plates	84
6.5 Nonlinear vibration of skew plates	89
6.6 Nonlinear vibration of column supported plates	92
6.7 Summary	98

## **CHAPTER: 7**

<b>NONLINEAR DYNAMIC INSTABILITY</b>	<b>99</b>
7.1 Introduction	99
7.2 Dynamic stability of plates by linear theory	99
7.3 Nonlinear forced vibration of plates under periodic in-plane load of the form $N_d \sin^2 \theta t$	102
7.4 Nonlinear forced vibration of plates under periodic in-plane load of the form $N_s + N_d \sin^2 \theta t$	105
7.5 Nonlinear forced vibration of plates under harmonic in-plane load $N_1 \cos 2\theta t$	108
7.6 Summary	112

## **CHAPTER: 8**

<b>NONLINEAR VIBRATION OF CYLINDRICAL PANELS</b>	<b>115</b>
8.1 Introduction	115
8.2 Validation and convergence study	115
8.3 Nonlinear free vibration of cylindrical panels	117
8.4 Nonlinear forced vibration of cylindrical panels	129
8.5 Summary	133

## **CHAPTER: 9**

<b>CONCLUSIONS</b>	<b>135</b>
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9.1 General .....	135
9.2 Major contributions .....	135
9.2.1 Plate and shell element .....	136
9.2.2 Stability analysis of Composite Plates .....	136
9.2.3 Large Amplitude Flexural Vibration of Composite Plates .....	137
9.2.4 Large amplitude vibration of cylindrical panels .....	139
9.3 Recommendations for future work .....	139
<b>REFERENCES .....</b>	<b>141</b>
<b>APPENDIX-A .....</b>	<b>155</b>
<b>A The plate bending elements .....</b>	<b>155</b>
<b>APPENDIX-B .....</b>	<b>159</b>
<b>B List of Publications Related to the Present Investigation .....</b>	<b>159</b>
<b>AUTHOR BIODATA .....</b>	<b>161</b>

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