

SOME STUDIES ON SIGNAL MATCHED FILTER BANKS

BINISH FATIMAH



**DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY DELHI
OCTOBER 2015**

© Indian Institute of Technology Delhi (IITD), New Delhi, 2015

SOME STUDIES ON SIGNAL MATCHED FILTER BANKS

by

BINISH FATIMAH

DEPARTMENT OF ELECTRICAL ENGINEERING

Submitted

in fulfillment of the requirements of the degree of

Doctor of Philosophy

to the



INDIAN INSTITUTE OF TECHNOLOGY DELHI
OCTOBER 2015

To my husband, parents and Prof. S.D. Joshi.

Certificate

This is to certify that the thesis entitled “**Some studies on signal matched filter banks**” being submitted by **Mrs. Binish Fatimah** to the **Department of Electrical Engineering, Indian Institute of Technology Delhi**, for the award of the degree of **Doctor of Philosophy** is the record of the bonafide research work carried out by her under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.

Date

New Delhi

(Dr. S. D. Joshi)

Professor

Department of Electrical Engineering

Indian Institute of Technology Delhi

Hauz Khas, New Delhi 110016

India

Acknowledgements

I would like to begin by thanking the Most Generous, the Most Merciful Lord Almighty for his immense blessings on me.

I am deeply grateful to my thesis supervisor Prof. Shiv Dutt Joshi. He is a research scholar's dream, he is not only a genius in his work but is an epitome of goodness and work ethics. His encouragement, positive attitude, sincere advice, constructive criticism kept me going when the going got tough. His attention to details made me go through certain topics multiple times and made me more dedicated than ever before. He is my definition of what success should seem like. I express my deepest thanks and regards to Mrs. Archana Joshi, for her motherly concern, care, love and support. It is always a pleasure to be in her company and her love makes me so indebted and at the same time emotional, it is hard to express my feelings in words.

I take this opportunity to thank Prof. Arun Kumar, Prof. Shankar Prakriya and Dr. Brejesh Lall for their support and valuable suggestions.

Parents, who sacrifice their sleep, comforts, interest and hobbies to see that we meet our dreams, who are our best admirers and loving critics, who make us feel like heroes even before we have established our worth in the world, who are an unending supply of love, motivation, warmth that we can always turn to. I thank them with all my heart.

I am indebted to my husband much more than any words can ever express. His selfless care was sometimes all that kept me going. He sacrificed his comfort, but made my dream his own and made this much more worthy of my time and effort, and gave me

a sense of achievement that would not have been possible without his constant support. It gives me great pleasure in acknowledging the understanding and love extended to me by my in-laws. They facilitated me to concentrate on my work even at the expense of family gatherings.

As said by Albert Schweitzer, “At times our own light goes out and is rekindled by a spark from another person. Each of us has cause to think with deep gratitude of those who have lighted the flame within us”. And I am so blessed to have a lot many people who did this for me, like my sisters Sana Swaliha, Hira Shamsi and my brother-in-law, Dr. Tauqeer Sarang, amazed me with their unconditional support and overflowing love that gave me warmth and comfort when I needed it the most.

My heartfelt thanks to Ms. Kanika Nadda, Ms. Garima Chauhan, Ms. Meena Krishnaia and Ms. Saba Hussain. They kept a sense of humour when I lost mine, shared their smiles with a generosity that made my days happier. They are my home away from home, on whom I can fall back with a trust of rock that I will be taken care of in hour of need. Ms. Garima Misra has this bottomless repository of love and care, and her home was opened to me. I was always welcomed there to get some warmth and comfort of home, when I was away from mine, I cannot thank her enough. Her thoughtfulness will always be cherished and remembered by me. I would also like to thank Ms. Sidra Khanam and Ms. Sanya Anees, for their support and hospitality during my stay in IIT Delhi.

I would also like to thank my colleagues from the research scholar lab, they have been a happy, productive and joyful group, which help to create a conducive environment to work.

Binish Fatimah

Abstract

Signals which possess features at multiple scales, either because of the generation process or because of the process of measurement, can be efficiently modeled using multiresolution modeling techniques. An accurate signal model could throw some light on the evolution mechanism of the underlying signal and can also provide its mathematical representation. The theory of multirate filter banks and wavelets have been exploited by the researchers to propose various multiscale modeling schemes, for both statistical and deterministic scenarios. Also, in the recent past signal dependent multiscale modeling techniques, based on signal adapted filter banks, have been explored. Signal adapted filter banks possess high energy compaction property and are thus used in a variety of signal processing applications like dimension reduction, communication, compression etc. Although different design techniques have been proposed to obtain these filter banks, a design strategy for general case, i.e. 2 channel/ M-channel/uniform/non-uniform and which leads to an FIR implementation is still an active area. Also computationally efficient solutions for the same are required to be found.

In view of the discussion above, we state the main issues, addressed in this thesis, are as follows: 1) given a signal or its statistics, obtain a signal adapted filter bank, which yields an optimum coding gain. Also provide a computationally efficient algorithm for the same, 2) assuming the class of signals which evolve in a multiband setting, model them as generated by synthesis side of the filter bank with stationary white noise inputs and obtain the model parameters and 3) given a signal or its statistics, propose a

signal dependent approach to decompose it into M-orthogonalized components carrying different spectral information.

To achieve these objectives, we begin with the concept of a signal matched filter bank, defined in [1], and propose a variate form which yields optimum coding gain. We also propose a geometric insight into the concept of signal matched filter bank and present a family of these filter banks with different design methodologies suitable for different applications.

A concept of signal matched whitening filter bank (SMWFB) is defined, which ensures de-correlation in time as well as across various channels, to yield an optimum coding gain. We obtain the required de-correlation by solving a constrained projection problem. The constrained projection problem is first proposed in a general context, in the form of a lemma. It is then used to define the required SMWFB.

The thesis also presents a concept of signal matched synthesis filter bank, corresponding to the proposed signal matched whitening filter bank, that reconstructs the given signal only. The multi input single output (MISO) synthesis filter bank is, first, converted into a single input single output (SISO) system, which is then interpreted in two different ways: first as a reconstruction filter for the proposed SMWFB and second as a model of generation for the given signal, which can be obtained as a solution to a prediction error problem.

A Hilbert space framework is defined to provide a geometrical interpretation of the proposed signal matched whitening filter bank as well as signal matched synthesis filter bank, for the “the given data case”. Using the geometric setting, we present a time and order recursive least squares algorithm for the proposed signal matched filter bank involving only scalar computations. The recursions, so obtained, can be put in a form of a lattice-like structure. The filter parameters are, however, not directly available from this structure. For the estimation of these parameters, order recursive algorithms are also presented. However, in some applications it may be more reasonable to assume

statistics rather than one realization. Thus, for such scenarios we have also developed a computationally efficient, order recursive algorithm for the proposed signal matched filter bank.

As a spin-off arising from the theory, developed in this thesis, we have proposed a signal matched filter bank to decompose a given signal into orthogonal components carrying different spectral information of the signal. Using the proposed scheme the signal can be decomposed into uniform as well as non-uniform frequency bands. Fast algorithms have also been developed to obtain the decomposition in a computationally efficient manner for “the given data case” as well as for “the given statistics case”. As another spin off, the concept of signal matched synthesis filter bank is re-worked to obtain a multi input multi output (MIMO) system identification framework, which can have an adaptive as well as block processing implementation. The algorithm developed, here, involves only scalar computations, unlike the conventional MIMO system identification algorithms, where one requires matrix computations.

Table of Contents

Certificate	i
Acknowledgements	ii
Abstract	iv
List of Figures	xi
List of Tables	xiv
List of Abbreviations and Symbols	xvi
1 Introduction	1
1.1 Motivation	1
1.2 Literature Survey	4
1.2.1 Signal Adapted Filter Banks	4
1.2.2 Signal Decomposition	6
1.3 Problem Definition and Scope of the Thesis	7
1.4 Organization of the thesis	10
2 Preliminaries	12
2.1 Notations Used	12
2.2 Multirate Filter Bank Theory: A brief overview	13

2.2.1	Basic Building Blocks	14
2.2.2	Noble Identities	17
2.2.3	Polyphase Decomposition	17
2.2.4	Perfect reconstruction filter bank	23
2.3	Some geometric results from the least squares estimation theory	23
2.4	Relevant results involving projection operator	25
2.5	Conclusions	27
3	Signal Matched Whitening Filter Bank: given data case	28
3.1	Introduction	28
3.2	Constrained projection problem	30
3.3	Signal Matched Whitening Filter Bank	32
3.3.1	Preliminaries	32
3.3.2	Geometric formulation and definition of Signal Matched Multi- rate Whitening Filter Bank	34
3.3.3	Observation	36
3.4	Development of the Algorithm	37
3.4.1	Notations and Preliminaries	37
3.4.2	Problem Statement	40
3.4.3	Development of the Algorithm	41
3.4.4	Complexity	43
3.5	Least Squares Estimation of the parameters of analysis filter bank	48
3.5.1	Complexity	51
3.6	Simulation Results	51
3.6.1	Coding gain	51
3.6.2	Convergence of filter bank parameters	54
3.6.3	Whitening the input spectra	55
3.7	Conclusions	58

4	Signal Matched Synthesis Filter Bank:given data case	59
4.1	Introduction	59
4.2	Signal Matched Synthesis Filter Bank	61
4.2.1	Preliminaries	61
4.2.2	Definition	65
4.2.3	Hilbert Space Framework	66
4.2.4	Problem Statement	67
4.2.5	Development of the algorithm	67
4.3	Least Squares Estimation of synthesis filter bank	72
4.3.1	Complexity	74
4.4	Simulation Results	74
4.5	Conclusions	78
5	Signal Matched Filter Bank with optimized coding gain: given statis-	
	tics case	80
5.1	Alternate representation for constrained projection problem	83
5.2	Signal Matched Whitening Filter Bank	86
5.2.1	Problem statement	87
5.2.2	Development of the algorithm	87
5.2.3	Complexity	95
5.2.4	Structure	95
5.3	Synthesis Filter Bank	98
5.3.1	Problem Statement	99
5.3.2	Development of Algorithm	100
5.3.3	Complexity	104
5.3.4	Structure	104
5.4	Simulations	106
5.5	Conclusions	107

6	Variate forms of signal matched filter bank	108
6.1	Signal decomposition using SMFB	109
6.1.1	Design philosophy	110
6.1.2	Design objectives	114
6.1.3	Design methodology	115
6.1.4	Levinson-type Algorithm for the “given statistics case”	117
6.1.5	Development of the Algorithm, “given data case”	121
6.1.6	Simulation Results	127
6.2	MIMO system identification	129
6.2.1	Interpreting a MIMO system as a SISO system	129
6.2.2	MIMO system identification using Signal Matched Synthesis Filter Bank	132
6.2.3	Simulation Results	134
6.3	Conclusions	135
7	Conclusions and Future works	137
7.1	Conclusions	137
7.2	Future Works	139
	References	139
	Appendix	150
A	Lemma 2 and Corollary 2	150
B	Circular Lattice	152
	Bio-data of the Author	156

List of Figures

1.1	Canonical decomposition of random process $x(t)$	2
2.1	M-channel multirate filter bank	13
2.2	M-fold Downsampler	14
2.3	M-fold Upsampler	15
2.4	Decimation Filter and a downsampler	16
2.5	Interpolation Filter and an upsampler	16
2.6	Noble Identities for Multirate systems	17
2.7	M-channel Analysis Filter Bank, in terms of type-I polyphase components	19
2.8	Synthesis Filters in terms of type-II polyphase components	21
2.9	Type-II polyphase decomposition of synthesis filter bank	22
2.10	Multirate Filter bank with polyphase matrices	22
3.1	M-channel multirate filter bank	32
3.2	Polyphase decomposition of analysis filter bank	33
3.3	M-forward linear predictors	34
3.4	Signal matched whitening filter bank	37
3.5	Lattice-ladder structure for SMWFB: given data case	47
3.6	Coding gain as ρ goes towards unity	53
3.7	Parameter trajectory of filter coefficients (a) bold line represents $h_0(1)$ and dashed line represents $h_0(2)$ and (b) the bold line is for $h_1(1)$ and the dashed line for $h_1(2)$	54

3.8	Parameter trajectory of filter coefficients (a) bold line represents $h_0(1)$ and dashed line represents $h_0(2)$ and (b) the bold line is for $h_1(1)$ and the dashed line for $h_1(2)$	55
3.9	Power spectral density of input signal(solid line), Output of first channel(dashed line) and output of second channel(dotted line) when the input signal, as given in Table 3.10 (a)signal 7, (b)signal 8 (c)signal 9(a)signal 7, (b)signal 8 (a)signal 7, (b)signal 8 and (c)signal 9.	57
4.1	Block diagram of the proposed signal matched filter bank	60
4.2	Polyphase decomposition of Synthesis filter bank	62
4.3	Single Input Single Output form of Signal Matched Synthesis Filter Bank	64
4.4	Lattice-Ladder structure for SMSFB: given data case	72
4.5	Input signal, is represented by solid line, and estimated signal, is represented by dotted line, when input is: (a) Signal 1, (b) Signal 4, (c) Signal 5, (d) Signal 6, (e) Signal 7 and (f)Signal 9, as given in Table 4.8.	76
4.6	Convergence of parameters (a) bold line represents $f_0(1)$ and dashed line represents $f_0(2)$ and (b) the bold line is for $f_1(1)$ and the dashed line for $f_1(2)$	77
5.1	An M-channel Analysis filter bank	81
5.2	Block diagram of the proposed SISO form of a synthesis filter bank . . .	82
5.3	Lattice-Ladder structure for a 3-channel SMWFB: given statistics case .	97
5.4	Lattice-Ladder structure for a 2-channel signal matched synthesis filter bank: given statistics case	105
5.5	Input signal, is represented by solid line, and the signal estimated using the algorithm from Table 5.1 and 5.2, is represented by dotted line, when input is (a)Signal 1 (b) Signal 2 and (c)Signal 3.	107
6.1	M-hannel multirate filter bank for signal decomposition	109

6.2	1 to M step ahead linear predictors	111
6.3	Orthogonalizing the filter outputs to obtain uniform frequency bands . .	112
6.4	Non-uniform frequency band obtained using method (i)	113
6.5	Signal Matched analysis filter for non-uniform case	113
6.6	Non-uniform frequency band obtained using method (ii)	114
6.7	Proposed signal matched analysis filter bank for the required signal de- composition	116
6.8	Lattice structure of the proposed signal mathced analysis filter bank . . .	127
6.9	Decomposition of a given signal, which is represented by “solid line”, output of channel 1 by “dashed line”, output of channel 2 by “dash-dot line” and output of channel 3 by “dotted line”	128
6.10	N-input M-output system	129
6.11	SISO form of the MIMO system	131
6.12	SISO interpretation of the MIMO system	131
6.13	MIMO system identification block diagram	132
B.1	Circular lattice	154

List of Tables

3.1	Definition of the auxiliary quantities arising in the LS algorithm for SMWFB , for $0 \leq i \leq M - 1$	44
3.2	Correlation coefficients in the LS algorithm for SMWFB	44
3.3	Substitution table for least squares algorithm of SMWFB	45
3.4	The LS algorithm for SMWFB: given data case	46
3.5	Substitution table for parameter estimation of SMWFB: given data case	50
3.6	The LS algorithm for parameter estimation of SMWFB: given data case	50
3.7	Coding Gain Comparison	52
3.8	Behavior of Coding gain (in dB) as number of channels change	53
3.9	Coding Gain Comparison between DCT and the proposed algorithm . . .	54
3.10	Input Signals For Experiment 5	56
4.1	Definition of auxiliary quantities arising in the algorithm developed for SMSFB: given data case, for $0 \leq i \leq M - 1$	69
4.2	Autocorrelation and cross-correlation coefficients used in the algorithm developed for SMSFB: given data case	70
4.3	Substitution table for LS algorithm for SMSFB:given data case	70
4.4	The LS algorithm for SMSFB: given data case	71
4.5	Definition of the auxiliary quantities arising in the parameter estimation algorithm of SMSFB: given data case	73

4.6	The substitution table for the parameter estimation algorithm of SMSFB: given data case, for $0 \leq i \leq M-1, 0 \leq j \leq M-1$	74
4.7	The LS parameter estimation algorithm of the SMSFB:given data case	74
4.8	Input Signals For Experiment 1	75
4.9	SMSFB actual parameters and estimated parameters, for $M=2$	78
5.1	Order recursive algorithm for SMWFB	94
5.2	Recursive algorithm for Signal Matched Synthesis Filter Bank	103
5.3	Comparing Coding Gain using the order recursive algorithm proposed for given statistics case	106
6.1	Order recursive algorithm for proposed signal matched analysis filter bank: given statistics case	119
6.2	Definition of the auxiliary quantities arising in the algorithm developed for the proposed analysis filter bank: given data case	124
6.3	Autocorrelation and cross-correlation coefficients	124
6.4	Substitution table for least squares algorithm of the proposed anlysis filter bank: given data case	125
6.5	The LS algorithm for the proposed analysis side: given data case	126
6.6	Parameter values of the first channel of SMSFB for different SNR	135
6.7	Parameter values of the second channel of SMSFB for different SNR	135
B.1	Circular Lattice Algorithm	153

List of Abbreviations and Symbols

AR	Autoregressive
ARMA	Autoregressive Moving Average
DWT	Discrete Wavelet Transform
FB	Filter Bank
FIR	Finite Impulse Response
IIR	Infinite Impulse Response
LHS	Left Hand Side
LPTV	Linear Periodically Time Varying
LS	Least Squares
LTI	Linear Time Invariant
MA	Moving Average
MIMO	Multi Input Multi Output
MISO	Multi Input Single Output
MMSE	Minimum Mean Square Error
NUFB	Non-Uniform Filter Bank

PCA	Principal Component Analysis
PCFB	Principal Component Filter Bank
PRFB	Perfect Reconstruction Filter Bank
PSD	Power Spectral Density
RHS	Right Hand Side
SIMO	Single Input Multi Output
SISO	Single Input Single Output
SMFB	Signal Matched Filter Bank
SMWFB	Signal Matched Whitening Filter Bank
SMSFB	Signal Matched Synthesis Filter Bank
SNR	Signal to Noise ratio
UFB	Uniform Filter Bank
WSCS	Wide Sense Cyclo-Stationary
WSS	Wide Sense Stationary