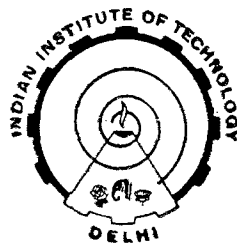


TRANSPORTATION TECHNIQUES IN FRACTIONAL AND RELATED PROGRAMMING PROBLEMS

by
PREM KUMAR SAXENA

Thesis submitted in fulfilment
of the requirements for the degree of
DOCTOR OF PHILOSOPHY



Department of Mechanical Engineering
INDIAN INSTITUTE OF TECHNOLOGY, DELHI

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R.S.

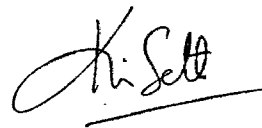
IN FOND MEMORY OF MY DAUGHTER

CERTIFICATE

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*TRANSPORTATION TECHNIQUES IN FRACTIONAL AND RELATED
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Mr Prem Kumar Saxena to the Indian Institute of
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Doctor of Philosophy, is a record of bonafide research
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and supervision and has fulfilled the requirements for
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This work has not been submitted elsewhere for the
award of any degree or diploma.



Dr Suresh Chandra
Assistant Professor
Department of Mathematics
Indian Institute of Technology
New Delhi 110016



Dr Kiran Seth
Assistant Professor
Department of Mechanical
Engineering
Indian Institute of
Technology
New Delhi 110016

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P.K. Saxena

(PREM KUMAR SAXENA)

ABSTRACT

Transportation within large and complex systems is itself a complex process involving the interaction of men and machines directed to the exchange/delivery of commodities among consumers within the complex system. This thesis is mainly devoted to the study and analysis of certain transportation techniques in fractional and related programming problems . Transportation problems with fractional objective functions arise in many real life situations of Management Science and Operations Research, where an individual, a group or a community is faced with the problem of maintaining good ratios between some very important crucial parameters concerned with the transportation of commodities .

An algorithm is developed to solve a linear fractional transportation problem with impurities in the commodity parameter. The algorithm developed to solve such real life problems of coal, and iron industries, depends heavily on the optimality conditions which have been derived here by using the fractional programming dual as well as by expressing the objective function in terms of the nonbasic variables.

A fractional transportation problem with impurities and excess total demand along with varying penalty costs for not meeting the individual demands is formulated and solved in linear plus the linear fractional programming set up.

A reoptimization procedure is developed for solving shipping cost ratio and shipping completion-date tradeoffs in a linear fractional transportation problem and also for shipment completion-date - total shipping cost tradeoffs in a fractional transportation problem with penalty costs.

Two different algorithms have been developed for solving impurity constrained transportation problem with respect to the time objective as well as cost-time objectives by relating them to a lexicographic transportation problem and solving by the use of lexicographic primal code.

A multi objective transportation problem having additional capacity and impurity restrictions is converted into a synthesis fractional objective function and using simplex multipliers technique, the optimality criteria are derived.

An initial efficient basic solution is determined for a multiple objective linear fractional transportation problem with additional impurity constraints by solving the related lexicographic minimum fractional transportation problem.

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