

LOCOMOTION OF FLEXIBLE SLENDER BODIES IN FLUID MEDIA

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LOCOMOTION OF FLEXIBLE SLENDER BODIES IN FLUID MEDIA

by

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*Dedicated to my family
for their endless love, support and encouragement*

Certificate

This is to certify that the thesis entitled “**Locomotion of flexible slender bodies in fluid media**”, being submitted by **Mr. Mohit Garg** to the Indian Institute of Technology Delhi for the award of the degree of **Doctor of Philosophy**, is a record of bonafide research carried out by him under my supervision. The thesis, in my opinion, merits evaluation in accordance with Institute’s rules and regulations. To the best of my knowledge and belief, the content of this thesis has not been submitted in full or in part to any other university or institute to confer any degree or diploma.

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Abstract

Fluid-structure interaction has been the subject of extensive study due to its application in numerous fields of scientific and applied research including the motion of micro-organisms, insects, birds, fishes, aircrafts, arterial blood flow, automotive aerodynamics, deflection of wind turbine blades, falling of a leaf, rocking motion of ships, elastic filaments, and flexible polymers in fluid media, to name a few. Broadly, this dissertation considers two topics in the context of fluid-structure interaction. The first part covers the theoretical derivation of a generalized slender body theory for the motion of special Cosserat filaments in Stokes flow. In the second part, an efficient singularity-free numerical technique is proposed to solve the motion of filaments in Stokes flow using the proposed slender body theory and further demonstrate the numerical technique through simple flow problems such as filament motion in background shear and compressional flow, and twirling-overwhirling motion of filament in background static flow.

To study the motion of filaments in fluid media, a well known *slender body theory* exists wherein the fluid flow is assumed to be Stokesian while the filament is modeled as a Kirchhoff rod which can bend and twist but remains inextensible and unshearable. In the present work, the inextensibility and unshearability constraints on filaments are relaxed, i.e., the filament is modeled as a special Cosserat rod. Starting with the boundary integral formulation of Stokes flow involving the filament's surface velocity and fluid traction that acts on the filament surface, the method of matched asymptotic expansion is used to first obtain a leading order representation of the boundary integral kernels in filament's aspect ratio. Thereafter, Fourier series expansion (in filament's circumferential coordinate) of both the filament's surface velocity and fluid traction are substituted in the aforementioned leading order representation. This is further linearized in the rod's shear strains to reduce the two-dimensional boundary integral over the filament surface into a line integral over the filament's centerline. Upon further collecting the coefficients of sine and cosine terms, the zeroth order Fourier mode yields a non-local line integral equation relating the rod's centerline velocity with the distributed fluid force that acts on the filament. The presence of line integral makes the relation non-local in nature. On the other hand, the first order Fourier mode yields a simpler local relation between the rod's angular velocity and the distributed fluid couple. The line integral equation is shown to reduce to the classical slender body theory when the shear strains and the axial strain are set to zero. The non-dimensional governing equations of the special Cosserat rod are also derived accounting for the distributed fluid force and distributed fluid couple in them.

As the derived slender body theory contains singularity, an efficient singularity free numerical scheme is proposed in the present work to solve the governing equations. It has been observed from the literature review that this singularity is tackled using cut-off

length, i.e., by removing the portion of singularity from the full domain or using mollifiers such as in the method of regularized stokeslet. In the present work, no cut-off length or mollifier is used to remove this singularity. It is demonstrated that the singularity is of removable type which, once removed, makes the system well defined in the entire domain. A second-order accurate finite-difference method is employed to discretize the governing equations. In addition to bending deformation of the filament, the present formulation also includes stretching, shearing and twisting deformations of the filament which have been largely neglected in existing literature. The numerical scheme is demonstrated through example problems of the filament behaviour in background shear flow wherein the importance of distributed couple term is investigated, background compressional flow wherein an interesting phenomenon of perversion is explored, and filament's twirling - overwhirling transitions when the filament is spun about its own axis in initially static fluid. The comparison between Kirchhoff and special Cosserat based slender body theories is performed in all the examples. It is shown that for relatively shorter filaments where the effect of shear and axial stretch is more dominant, the results obtained from the present work deviate from the ones based on the classical slender body theory.

सारांश

द्रव-संरचना अंतरक्रिया वैज्ञानिक और अनुप्रयोगिक अनुसंधान के कई क्षेत्र जिसमें उदाहरण के रूप में सूक्ष्मजीवों, कीटाणुओं, पक्षियों, मछलियों, एयरक्राफ्ट, धमनी रक्त प्रवाह, ऑटोमोटिव एयरोडायनेमिक्स, वायु चक्रवात, पंखों की विकृति, पत्तियों का गिरना, जहाजों की हिलने की गति, लचीले धाराओं में एलास्टिक फिलामेंट, और द्रव्य माध्यम में लचीले पॉलिमर्स को लिया जा सकता है, में अपने प्रयोग के कारण विस्तार से अध्ययन का विषय बनाया है। विस्तृत रूप से, यह प्रकरण द्रव-संरचना अंतरक्रिया के संदर्भ में दो प्रसंगों को ध्यान में लेता है। पहला भाग स्टोक्स फ्लो में विशेष कॉसेरेट फिलामेंट के चलन के लिए एक सामान्यकृत स्लेंडर बॉडी सिद्धांत के सिद्धांतिक निर्माण को कवर करता है। दूसरे हिस्से में, एक कुशल सिंग्युलैरिटी-मुक्त संख्यात्मक तकनीक प्रस्तुत किया गया है, जिसमें स्लेंडर बॉडी सिद्धांत का उपयोग करके स्टोक्स फ्लो में विशेष कॉसेरेट फिलामेंट के चलन को हल किया गया है, और इस संख्यात्मक तकनीक को सरल फ्लो समस्याएँ जैसे कि पृष्ठभूमि कतरनी और संपीड़नात्मक प्रवाह में फिलामेंट की गति तथा फिलामेंट का स्थिर पृष्ठभूमि फ्लो में चक्रण-अतिआवर्तन चलन द्वारा प्रदर्शित किया गया है।

धाराओं के गतिविधि का अध्ययन करने के लिए, एक प्रसिद्ध स्लेंडर बॉडी सिद्धांत मौजूद है जिसमें द्रव प्रवाह को स्टोक्सियन माना जाता है जबकि फिलामेंट को एक किरचोफ रॉड के रूप में मॉडल किया जाता है जो ट्विस्ट हो सकता है और बेंड हो सकता है लेकिन स्ट्रेच और शीयर नहीं हो सकता। वर्तमान काम में, फिलामेंट्स पर अस्ट्रेचेबिलिटी और अंशियरेबलता की बाधाएं कमजोर की गई हैं, अर्थात् फिलामेंट को एक विशेष कॉसेरेट रॉड के रूप में मॉडल किया गया है। स्टोक्स प्रवाह के बाउंडरी इंटीग्रल फॉर्मूलेशन से शुरू होकर, जिसमें फिलामेंट के सतह वेग और द्रव कर्षण शामिल हैं (जो फिलामेंट की सतह पर कार्य करता है), मिलान किए गए एसिम्प्टोटिक विस्तार की विधि का उपयोग पहले बाउंडरी इंटीग्रल कर्नेल के एक लीडिंग ऑर्डर प्रतिनिधित्व (फिलामेंट के आस्पेक्ट रेशियो में) को प्राप्त करने के लिए किया जाता है। इसके बाद, फिलामेंट की सतही वेग और फिलड टैक्शन के फुरीये श्रृंगी विस्तार (फिलामेंट के परिधि निर्देशांक में) को ऊपर बताये गए लीडिंग ऑर्डर प्रतिनिधित्व में डाला गया है। इसे फिर रॉड के शीयर स्ट्रेन में लीनियरीकृत किया जाता है ताकि फिलामेंट सतह के ऊपर दो-आयामी बाउंडरी इंटीग्रल की जगह फिलामेंट की केंद्रेखा के ऊपर का लाइन इंटीग्रल पाया जा सके। साइन और कोसाइन शब्दों के कोएफिशिएंट्स को एकत्रित करने पर, शून्यवादी फोरिए मोड एक गैर-स्थानीय रेखांतर समीकरण पैदा करता है जिसमें रॉड की केंद्रेखा वेग को फिलामेंट पर कार्रवाई करने वाले वितरित फिलड बल के साथ संबंधित किया गया है। रेखांतर समीकरण के उपस्थित होने के कारण यह संबंध प्राकृतिक रूप में गैर-स्थानीय है। दूसरी ओर, पहले क्रम का फोरिए मोड रॉड की कोणीय गति और वितरित फिलड कपल के बीच में एक सरल स्थानीय संबंध पैदा करता है। यह दिखाया गया है कि शीयर स्ट्रेन और अक्षीय स्ट्रेन को शून्य करने पर लाइन इंटीग्रल समीकरण क्लासिकल स्लेंडर बॉडी थ्योरी बन जाता है। विशेष कॉसेरेट रॉड के गैर-आयामी गवर्निंग समीकरण, वितरित फिलड बल और वितरित फिलड कपल को ध्यान में रखते हुए, भी निकाले गए हैं।

क्योंकि निकाले गए स्लेंडर बॉडी सिद्धांत में एक अपूर्वता है, वर्तमान काम में गवर्निंग समीकरणों को हल करने के लिए एक कुशल अपूर्वता रहित संख्यात्मक योजना प्रस्तुत की गई है। साहित्य समीक्षा से पाया गया है कि इस अपूर्वता को कट-ऑफ लंबाई, अर्थात्, पूर्ण डोमेन से अपूर्वता का हिस्सा निकालकर, या मॉलिफायर का उपयोग करके (मैथड ऑफ रेग्युलराइज्ड स्टोक्सलेट की तरह) हल किया गया है।

वर्तमान काम में, इस अपूर्वता को हटाने के लिए कोई कट-ऑफ लंबाई या मॉलिफायर का उपयोग नहीं गया है। यह दिखाया गया है कि यह अपूर्वता हटाने योग्य प्रकार का है जो, एक बार हटा दिया जाए, तो पूरे डोमेन में प्रणाली को पूरी तरह परिभाषित बना देता है। गवर्निंग समीकरणों को डिस्क्रेटाइज़ करने के लिए एक द्वितीयक ऑर्डर परिमित अंतर तकनीक का उपयोग किया जाता है। फिलामेंट की बेन्डिंग डेफोर्मेशन के अलावा, वर्तमान सूत्राणि में इस समीकरण में फिलामेंट की स्ट्रेचिंग, शियरिंग और ट्विस्टिंग डेफोर्मेशन को भी शामिल किया गया है जिन्हें मौजूदा साहित्य में बड़े ही अनदेखे गए हैं। इस संख्यात्मक तकनीक को पृष्ठभूमि कतरनी प्रवाह में फिलामेंट के व्यवहार जिसमें वितरित बल युग्म शब्द के महत्व की जाँच की गई है, पृष्ठभूमि संपीड़नात्मक प्रवाह जिसमें विकृति की दिलचस्प घटना का अन्वेषण किया गया है, तथा फिलामेंट के चक्रण-अतिआवर्तन बदलाव जब फिलामेंट को शांत द्रव में अपने धुर के चारों ओर घुमाया गया है के उदाहरण समस्याओं के माध्यम से प्रदर्शित किया गया है। किरचॉफ और विशेष कोसैरैट आधारित पतले शरीर सिद्धांतों के बीच तुलना सभी उदाहरणों में की गई है। यह दिखाया गया है कि अपेक्षाकृत छोटे फिलामेंट्स के लिए, जहां कतरनी और और अक्षीय खिंचाव का प्रभाव अधिक प्रभावशाली होता है, वर्तमान काम से प्राप्त परिणाम क्लासिकल स्लेंडर बॉडी सिद्धांत पर आधारित परिणामों से भिन्न हैं।

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