# STATE SPACE ROUTH APPROXIMANTS AND THEIR APPLICATIONS

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# ABSTRACT

Applications of control techniques to large engineering systems generally lead to mathematical system descriptions which are complex and of high dimension and thereby present difficulties in the analysis, and design. One of the ways of taking care of such complex systems is to consider simplified models of the system or of suitably selected portions of the system. A large number of methods of simplification procedures are available for deriving reduced dimensional models of linear systems both in the time as well as in frequency domain. Amongst them a frequency domain method based on Routh approximants is reported to possess many desirable features. However, for employing most of the recent concepts of control theory and system analysis via reduced order models, one needs reduced dimensional models in the time domain. It was, therefore, proposed to extend the Routh approximant technique for model simplification to time domain.

A transfer function expansion called $\gamma$-$\delta$ expansion is proposed. The choice of the $\gamma$-$\delta$ expansion and its time domain realization yield lower dimensional state space models by suitable truncation for single input single output systems. For multivariable system an exact realization is not known and

the lower dimensional models are derived by truncation under relaxed conditions, making use of an optimum transformation matrix. Controllability properties of reduced order models have been examined and a procedure is developed for deriving lower dimensional models for systems which are not completely state controllable.

In general, the various reduced order modelling procedures do not indicate any set criterion for determining the optimum order of the model which can approximate the system adequately. This weakness of the model reduction has been removed in the proposed Routh approximant method and a criterion valid for scalar as well as multivariable systems is proposed in this thesis.

In some situations like missile and aircraft control and chemical processes the system under control may be inherently unstable and is usually of relatively high complexity. Simplified models based on the $\gamma$-$\delta$ expansion, are derived for such unstable systems and cover all types of instability. A state space Routh approximant technique has also been proposed for reducing high order time varying systems of a particular class to time invariant lower order systems.

In certain control system design procedures based on lower dimensional models a time domain relation, between the high order system state vector and lower dimensional model state vector, in the form of an aggregation matrix proves useful. A relationship between the state vectors of the system and the model is derived for the proposed simplification procedures and is referred to as an 'approximate aggregation matrix'.

The application of the proposed Routh approximant models is demonstrated for three situations. The problem of reduction of control law complexity and design of a suboptimal regulator are considered first. Derivation of dynamic equivalents for an interconnected power system is discussed next. A simplification of individual portions of a power system is done and a suitable connection of these simplified models yield the desired dynamic equivalents.

An extension of the Routh approximant procedure for handling a certain type of distributed parameter systems is included in an appendix.

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