

STATE SPACE ROUTH APPROXIMANTS AND THEIR APPLICATIONS

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ABSTRACT

Applications of control techniques to large engineering systems generally lead to mathematical system descriptions which are complex and of high dimension and thereby present difficulties in the analysis, and design. One of the ways of taking care of such complex systems is to consider simplified models of the system or of suitably selected portions of the system. A large number of methods of simplification procedures are available for deriving reduced dimensional models of linear systems both in the time as well as in frequency domain. Amongst them a frequency domain method based on Routh approximants is reported to possess many desirable features. However, for employing most of the recent concepts of control theory and system analysis via reduced order models, one needs reduced dimensional models in the time domain. It was, therefore, proposed to extend the Routh approximant technique for model simplification to time domain.

A transfer function expansion called γ - δ expansion is proposed. The choice of the γ - δ expansion and its time domain realization yield lower dimensional state space models by suitable truncation for single input single output systems. For multivariable system an exact realization is not known and

the lower dimensional models are derived by truncation under relaxed conditions, making use of an optimum transformation matrix. Controllability properties of reduced order models have been examined and a procedure is developed for deriving lower dimensional models for systems which are not completely state controllable.

In general, the various reduced order modelling procedures do not indicate any set criterion for determining the optimum order of the model which can approximate the system adequately. This weakness of the model reduction has been removed in the proposed Routh approximant method and a criterion valid for scalar as well as multivariable systems is proposed in this thesis.

In some situations like missile and aircraft control and chemical processes the system under control may be inherently unstable and is usually of relatively high complexity. Simplified models based on the γ - δ expansion, are derived for such unstable systems and cover all types of instability. A state space Routh approximant technique has also been proposed for reducing high order time varying systems of a particular class to time invariant lower order systems.

In certain control system design procedures based on lower dimensional models a time domain relation, between the high order

system state vector and lower dimensional model state vector, in the form of an aggregation matrix proves useful. A relationship between the state vectors of the system and the model is derived for the proposed simplification procedures and is referred to as an 'approximate aggregation matrix'.

The application of the proposed Routh approximant models is demonstrated for three situations. The problem of reduction of control law complexity and design of a suboptimal regulator are considered first. Derivation of dynamic equivalents for an interconnected power system is discussed next. A simplification of individual portions of a power system is done and a suitable connection of these simplified models yield the desired dynamic equivalents.

An extension of the Routh approximant procedure for handling a certain type of distributed parameter systems is included in an appendix.

CONTENTS

	Page
List of Figures	i
List of Tables	iii
Abstract	iv
CHAPTER	
1 INTRODUCTION	1
2 REVIEW OF RELATED LITERATURE	8
2.1 Dominant Eigenvalue Approach	9
2.2 Model Simplification via Aggregation	15
2.3 Pade Approximation Method	17
2.4 Routh Approximation Technique	21
2.4.1 Frequency Domain Approach	21
2.4.2 State Space Approach	26
2.4.3 Controllability and Observability Concepts	31
2.5 Reduced Order Models via Phase Canonical Forms	33
2.6 Simplification of Unstable Systems	37
2.7 Remarks	41
3 STATE SPACE ROUTH APPROXIMANTS FOR SCALAR SYSTEMS	44
3.1 Introduction	44
3.2 γ - δ Expansion	45

CONTENTS (CONTD.)

CHAPTER		Page
	3.3 Model Simplification in Frequency Domain	55
	3.3.1 Stability of the Simplified Model	60
	3.3.2 Numerical Example	60
	3.3.3 Realization of γ - δ Canonic form	65
	3.4 Simplified Models in State Space	66
	3.4.1 γ - δ Description in State Space	66
	3.4.2 Even and Odd Order Reduced Models	72
	3.4.3 Approximate Aggregation Matrix	78
	3.5 Numerical Example	82
	3.6 Choice of Model Order	87
	3.7 Remarks	91
4	ROUTH APPROXIMANT METHOD FOR MULTIVARIABLE SYSTEMS	97
	4.1 Introduction	97
	4.2 γ - δ Expansion	98
	4.3 Model Simplification in Frequency Domain	102
	4.4 Model Simplification in Time Domain	105
	4.4.1 Single Input Multi Output Case	105
	4.4.2 Multi Input Multi Output Case	107
	4.4.3 Numerical Example	110
	4.5 System with Uncontrollable Modes	113
	4.6 Choice of Model Order	119
	4.7 Remarks	123

CONTENTS (CONTD.)

CHAPTER		Page
5	ROUTH APPROXIMANT METHOD FOR UNSTABLE SYSTEMS AND TIME VARYING SYSTEMS	129
5.1	Introduction	129
5.2	Simplification of Unstable System via Expansion	130
5.2.1	System with Simple Pole at the Origin	131
5.2.2	System with Multiple Poles at the Origin	132
5.2.3	System with a Complex Extremal Pole	137
5.2.4	A General Case	139
5.3	Simplification of Unstable System in State Space	141
5.3.1	System with Extremal Positive Real Eigenvalue	142
5.3.2	System with Extremal Complex Eigenvalues	145
5.3.3	System with Multiple Zero Eigenvalues	151
5.4	Simplification of Time Varying Systems	157
5.4.1	Transformation of a Class of Autonomous Time Varying Systems to Time Invariant Systems	158
5.4.2	Transformation of a Class of non Autonomous Time Varying Systems to Time Invariant Systems	160

CONTENTS (CONTD.)

CHAPTER		Page
	5.4.3 Simplification of a Class of non Autonomous Time Varying Systems	163
	5.5 Remarks	169
6	APPLICATIONS OF STATE SPACE ROUTH APPROXIMANTS	176
	6.1 Introduction	176
	6.2 Routh Approximants for Suboptimal Control	177
	6.2.1 Control Law Reduction	178
	6.2.2 Regulator Design via Routh Approximants	186
	6.3 Routh Approximant Dynamic Equivalents for Power System Stability Analysis	190
	6.3.1 Single Machine Connected to Infinite Bus	192
	6.3.2 Two Machine System Connected to Infinite Bus	197
	6.3.3 Numerical Example	202
7	CONCLUSIONS	210
	7.1 Summary of the Principal Results	210
	7.2 Suggestion for further Work	213
	REFERENCES	215
	Appendix A Certain Transformation Matrices	223
	Appendix B Application of Routh Approximants for Discrete System and Distribu- ted Parameter System	237