

# **CONTROL STRATEGIES FOR ACTIVE MAGNETIC BEARING SYSTEM**

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**CONTROL STRATEGIES FOR ACTIVE MAGNETIC  
BEARING SYSTEM**

*by*

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*to the*



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# CERTIFICATE

This is to certify that the thesis entitled “ **Control Strategies for Active Magnetic Bearing System**”, submitted by **Sudipta Saha** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in Electrical Engineering, is a record of the bonafide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the award of the degree.

The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

Dr. M. Nabi

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(Supervisor)

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*Sudipta Saha*

Sudipta Saha

# ABSTRACT

The rotating machines often have to deal with the deviations of the shaft due to the rotor dynamics. These deviations are caused by different reasons, like nonlinear electromagnetic forces, external disturbance forces, gyroscopic effects, coupling and system parameter variations etc. The active magnetic bearing (AMB) is employed to support the shaft in the air and control the displacements in the rotors. The applications of AMB in industries have been expanded due to the various advantages over traditional mechanical bearings. Some of the notable advantages are less energy utilization, high speed rotation, reduced friction losses, reduced vibration and noise, higher longevity, etc. The aim of this thesis is to present a comprehensive idea on the modeling and controller design for distributed and lumped parameter AMB models. For distributed parameter AMB system, a robust control strategy like Sliding mode control (SMC) is proposed. Furthermore, different robust and optimal control strategies are proposed for lumped parameter AMB model.

The modeling and control of AMB system is a complex task. Modeling can be classified into two aspects: Analytical modeling and Numerical modeling. In numerical modeling, the model is developed using partial differential equations, but to solve PDE is a complex task. The other option is to discretize the PDEs with the help of discretization method, which will give large numbers of Ordinary Differential Equations (ODE's). But FE models are generally large, involving large numbers of ordinary equations and variables, makes their solution a time consuming process. Hence, a lower dimensional computational model is also known as Reduced Order Model (ROM), need to be generated. This conversion can be done by various Model Order Reduction (MOR) techniques based on eigenvalue decomposition, balance truncation, Krylov subspace, etc.

Controller design for AMBs is another challenge. However, in this thesis, sliding mode control (SMC), has been used to design controllers for distributed AMB model, where the

ROM is used as an observer. Further in SMC, the best solution to alleviate the chattering phenomenon, without losing the invariance property of sliding mode against uncertainties, is the use of higher-order SMC (HOSMC) techniques. This thesis proposes the design of higher-order sliding mode control of the lumped AMB system, considering limited or no knowledge of the upper bound of the disturbances, where the finite time convergence is achieved. This was motivated by the desire for designing adaptive integral second order and third order SMC for AMB system. To reduce the chattering phenomenon in SMC, Fuzzy sliding mode inspired control (FSMIC) is also designed, where the bound of the lumped uncertainty is known in advance.

Furthermore in this thesis, a relatively new numerical optimal control method called the Pseudospectral method (PSM) has been used for minimizing the time-energy consumption of the nonlinear AMB system while driving the states to their desired end point conditions. PSM converts nonlinear optimal control problem to nonlinear programming problem (NLP) by discretizing the cost function, states and controls with the help of orthogonal functions. Also, the method is easy to implement even for nonlinear systems, and gives relatively fast rate of convergence. Finally, the last chapter describes a summary of thesis contributions along with the pros and cons of different control techniques and future directions of the research for AMB system.

**Keywords:** Active magnetic bearing, Sliding mode control, Distributed parameter system, Lumped parameter system, Higher order sliding mode control, Fuzzy logic, Pseudospectral method.

## सार

घूर्णन मशीनों को अक्सर रोटार के कारण शाफ्ट के विचलन से निपटना पड़ता है गतिकी। ये विचलन विभिन्न कारणों से होते हैं, जैसे नॉनलाइनियर इलेक्ट्रोमैग्नेटिक बल, बाहरी अशांति बल, जाइरोस्कोपिक प्रभाव, युग्मन और सिस्टम पैरामीटर विविधताएं पैरामीटर आदि। सक्रिय चुंबकीय असर (एएमबी) को हवा और नियंत्रण में शाफ्ट का समर्थन करने के लिए नियोजित किया जाता है रोटार में विस्थापन। उद्योगों में एएमबी के अनुप्रयोगों का विस्तार किया गया है पारंपरिक यांत्रिक बीयरिंगों पर विभिन्न लाभों के लिए। कुछ उल्लेखनीय फायदे कम ऊर्जा उपयोग, उच्च गति रोटेशन, कम घर्षण नुकसान, कम कंपन और शोर, उच्च दीर्घायु, आदि। इस थीसिस का उद्देश्य पर एक व्यापक विचार प्रस्तुत करना है वितरित और लम्बु पैरामीटर एएमबी मॉडल के लिए मॉडलिंग और नियंत्रक डिजाइन। के लिये वितरित पैरामीटर एएमबी सिस्टम, स्लाइडिंग मोड कंट्रोल (एसएमसी) जैसी एक मजबूत नियंत्रण रणनीति प्रस्ताव है। इसके अलावा, विभिन्न मजबूत और इष्टतम नियंत्रण रणनीतियों का प्रस्ताव है गांठदार पैरामीटर एएमबी मॉडल।

एएमबी प्रणाली का मॉडलिंग और नियंत्रण एक जटिल कार्य है। मॉडलिंग को वर्गीकृत किया जा सकता है दो पहलुओं में: विश्लेषणात्मक मॉडलिंग और संख्यात्मक मॉडलिंग। संख्यात्मक मॉडलिंग में, मॉडल को आंशिक अवकल समीकरणों का उपयोग करके विकसित किया गया है, लेकिन पीडीई को हल करना एक जटिल कार्य है। दूसरा विकल्प पीडीई को विवेकीकरण विधि की मदद से अलग करना है, जो होगा बड़ी संख्या में साधारण अंतर समीकरण (स्तोत्र) दें। लेकिन एफई मॉडल आम तौर पर होते हैं बड़े, बड़ी संख्या में साधारण समीकरणों और चरों को शामिल करते हुए, उनके समाधान को एक समय बनाते हैं उपभोग प्रक्रिया। इसलिए, एक निम्न आयामी कम्प्यूटेशनल मॉडल को रिड्यूस्ड के रूप में भी जाना जाता है ऑर्डर मॉडल (रोम), जेनरेट करने की जरूरत है। यह रूपांतरण विभिन्न मॉडल द्वारा किया जा सकता है ऑर्डर रिडक्शन (एमओआर) तकनीक ईजेनवैल्यू डीकंपोजिशन, बैलेंस ट्रंकेशन पर आधारित है, क्रायलोव सबस्पेस, आदि।

एएमबी के लिए नियंत्रक डिजाइन एक और चुनौती है। हालाँकि, इस थीसिस में, स्लाइडिंग मोड नियंत्रण (एसएमसी), का उपयोग वितरित एएमबी मॉडल के लिए नियंत्रकों को डिजाइन करने के लिए किया गया है, जहां रोम एक पर्यवेक्षक के रूप में प्रयोग किया जाता है। इसके अलावा एसएमसी में, बकबक को कम करने का सबसे अच्छा समाधान घटना, अनिश्चितताओं के खिलाफ स्लाइडिंग मोड की अपरिवर्तनीय संपत्ति को खोए बिना, है उच्च-क्रम वाली एसएमसी (एचओएसएमसी) तकनीकों का उपयोग। यह थीसिस उच्च के डिजाइन का प्रस्ताव करती है सीमित या कोई ज्ञान नहीं होने पर, लुम्बु एएमबी सिस्टम के स्लाइडिंग मोड नियंत्रण का आदेश दें विक्षोभ की ऊपरी सीमा, जहां परिमित

समय अभिसरण प्राप्त किया जाता है। यह अनुकूली अभिन्न दूसरे क्रम और तीसरे क्रम के एसएमसी को डिजाइन करने की इच्छा से प्रेरित था एएमबी सिस्टम के लिए एसएमसी में बकबक की घटना को कम करने के लिए, फ़ज़ी स्लाइडिंग मोड प्रेरित नियंत्रण (एफएसएमआईसी) को भी डिज़ाइन किया गया है, जहाँ लम्ड अनिश्चितता की सीमा को जाना जाता है अग्रिम।

इसके अलावा इस थीसिस में, एक अपेक्षाकृत नई संख्यात्मक इष्टतम नियंत्रण विधि जिसे . कहा जाता है समय-ऊर्जा की खपत को कम करने के लिए स्यूडोस्पेक्ट्रल विधि (पीएसएम) का उपयोग किया गया है राज्यों को उनकी वांछित अंतिम बिंदु की स्थिति में ले जाते समय नॉनलाइनियर एएमबी प्रणाली का। पीएसएम नॉनलाइनियर इष्टतम नियंत्रण समस्या को नॉनलाइनियर प्रोग्रामिंग समस्या (एनएलपी) में परिवर्तित करता है ऑर्थोगोनल फ़ंक्शंस की मदद से लागत फ़ंक्शन, राज्यों और नियंत्रणों का विवेक करना। भी, गैर-रेखीय प्रणालियों के लिए भी विधि को लागू करना आसान है, और अपेक्षाकृत तेज दर देता है अभिसरण। अंत में, अंतिम अध्याय विभिन्न नियंत्रण तकनीकों के पेशेवरों और विपक्षों के साथ थीसिस योगदान का सारांश और एएमबी प्रणाली के लिए अनुसंधान के भविष्य के निर्देशों का वर्णन करता है।

कीवर्ड: सक्रिय चुंबकीय असर, स्लाइडिंग मोड नियंत्रण, वितरित पैरामीटर सिस्टम, लम्ड पैरामीटर सिस्टम, हायर ऑर्डर स्लाइडिंग मोड कंट्रोल, फ़ज़ी लॉजिक, स्यूडोस्पेक्ट्रल तरीका।

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# List of Abbreviations

<b>Abbreviations</b>	<b>Description</b>
AMB	Active Magnetic Bearing
FEM	Finite Element Modeling
FVM	Finite Volume Method
FDM	Finite Difference Method
BEM	Boundary Element Method
MOR	Model Order Reduction
ROM	Reduced Order Model
FOM	Full Order Model
BT	Balanced Truncation
PID	Proportional Integral Derivative
DOF	Degrees of Freedom
SMC	Sliding Mode Control
TAMB	Thrust Active Magnetic Bearing
RAMB	Radial Active Magnetic Bearing
SOSMC	Second Order Sliding Mode Control
HOSMC	Higher Order Sliding Mode Control
AISOSMC	Adaptive Integral Second Order Sliding Mode Control
AITOSMC	Adaptive Integral Third Order Sliding Mode Control

*Continued from previous page*

<b>Abbreviations</b>	<b>Description</b>
FSMIC	Fuzzy Sliding Mode Inspired Control
VSCS	Variable Structure Control Systems
PSM	Pseudospectral Method
LPSM	Legendre Pseudospectral Method
CPSM	Chebyshev Pseudospectral Method
LTl	Linear Time Invariant
MIMO	Multi Input Multi Output
PDE	Partial Differential Equation
ODE	Ordinary Differential Equation
DAE	Differential Algebraic Equation
LQR	Linear Quadratic Regulator
LQG	Linear Quadratic Gaussian
NLP	Nonlinear Programming
FLC	Fuzzy Logic Control
GL	Gauss Lobato
C.G.	Center of Gravity
M.E.S.S	Matrix Equation Sparse Solver
RMSE	Root Mean Square Error
TVs	Total Variations

# List of Symbols

List of Symbols	Description
$(\cdot)^T$	General notation for the matrix transpose operation
$\mathbb{R}$	The field of real numbers
$\mathbb{R}^n$	The $n$ -dimensional real vector space
$s$	Sliding surface
$u_{eq}$	Equivalent control input
$u_{sw}$	Switching control input
$I_y$	Control current
$I_0$	Bias current
$N$	Number of turns
$H$	Magnetic field intensity
$l$	Length of AMB
$B$	Magnetic Flux Density
$\mu_0$	Permeability of free space
$\mu_r$	Permeability of medium
$\phi$	Flux
$W$	Total energy
$F$	Electromagnetic Force
$V$	Volume

<b>List of Symbols</b>	<b>Description</b>
$S$	Cross sectional area
$B_0$	Magnetic flux density due to $I_0$
$B_y$	Magnetic flux density due to $I_y$
$A_0$	Magnetic vector potential due to $I_0$
$A_y$	Magnetic vector potential due to $I_y$
$\hat{A}_y$	Nodal values of $A_y$
$\hat{F}_y$	Input to FEM Model
$J_0$	Current density due to $I_0$
$J_y$	Current density due to $I_y$
$\sigma$	Electrical conductivity
$m$	Mass
$g$	Gravitational constant
$t$	Time
$m$	Number of inputs in system
$\bar{E}, \bar{A}, \bar{B}, \bar{C}$	State space matrix of LTI system
$V, W$	Projection matrices
$q$	Size of reduced order model
$\bar{E}_q, \bar{A}_q, \bar{B}_q, \bar{C}_q$	Reduced Model
$G$	Transfer function
$K_{q1}$	Input Krylov subspsce
$K_{q2}$	Output Krylov subspsce
$W_c$	Controllability Grammian
$W_o$	Observability Grammian
$\Delta_A$	Sum of all unmodeled dynamics

**List of Symbols****Description**

$\delta$	Uncertainty due to neglected parts
$\mathcal{L}^{-1}$	Inverse Laplace transform
$\bar{w}$	Upper bound of uncertainty
$\eta$	Positive design constant
$\mathcal{V}_1$	Lyapunov function candidate
$\lambda_{max}(A)$	Maximum eigenvalue of matrix $A$
$\lambda_{min}(A)$	Minimum eigenvalue of matrix $A$
$\mathbf{e}$	Error
$\mathbf{M}$	Mass matrix
$\mathbf{A}$	Stiffness matrix
$\mathbf{B}$	Control gain matrix
$\mathbf{x}$	State vector
$\mathbf{u}$	Control input vector
$\mathbf{a}$	Distance between C.G & left RAMB
$\mathbf{b}$	Distance between C.G & right RAMB
$\mathbf{c}$	Distance between C.G & end of rotor
$\mathbf{J}$	Transverse moment of inertia
$\mathbf{J}_z$	Polar moment of inertia
$f_{dtx}, f_{dty}, f_{dtz}$	Disturbance forces
$\mathcal{T}_f$	Final time
$\mathcal{T}_0$	Initial time
$\mathbf{A}_n$	Nominal part of matrix $\mathbf{A}$
$\mathbf{B}_n$	Nominal part of matrix $\mathbf{B}$
$\Delta A, \Delta B$	Model uncertainty

<b>List of Symbols</b>	<b>Description</b>
$\mathbf{d}$	Lumped uncertainty
$\phi_n$	Nominal control input
$\phi_{dis}$	Discontinuous control input
$G_1, G_2, G_3$	Design Constants
$\text{sig}(\cdot)$	Signum function
$\mathbf{v}(t)$	Auxiliary control input
$k_{ti}$	Current stiffness for thrust AMB
$k_{tp}$	Position stiffness for thrust AMB
$k_{fi}$	Current stiffness for radial AMB
$k_{fp}$	Position stiffness for radial AMB
$\hat{\alpha}, \hat{\beta}$	Adaptive parameter
$K_p$	Proportional gain
$K_i$	Integral gain
$K_d$	Derivative gain
$\mathbf{G}_q$	Auxiliary control gain
$x_b, y_b, z_b$	Nominal air gap position
$\mathbf{x}_c, \mathbf{y}_c, \mathbf{z}_c, \theta_x, \theta_y$	C.G points
$\omega$	Regulation speed
$\mathbf{R}$	Weight matrix
$\mathcal{J}$	Objective function
$\mathcal{M}$	End point cost
$\mathcal{L}$	Running cost
$\mathcal{P}_N$	Legendre polynomial of degree $N$
$\varphi_n, \xi_n$	Lagrange polynomial

**List of Symbols****Description** $\mathcal{D}_{k,n}$ 

Differentiation matrix

 $\mathcal{W}_k$ 

Quadrature weights

 $\mathcal{T}_N$ Chebyshev polynomial of  $N^{\text{th}}$  order