

MATHEMATICAL MODELING OF THREE ZONE SOLAR PONDS—THERMAL AND STABILITY ANALYSES

BY
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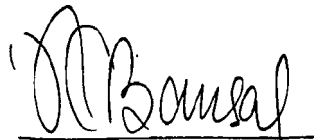
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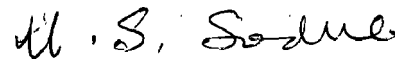
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C E R T I F I C A T E

This is to certify that the thesis entitled
"Mathematical Modeling of Three Zone Solar Ponds - Thermal
and Stability Analyses" being submitted by Ms. Yojana Katti
to the Indian Institute of Technology, Delhi, for the award
of the degree of Doctor of Philosophy, is a record of bona-
fide research work carried out under our supervision. The
results contained in this thesis have not been submitted
to any other University or Institute for the award of any
degree or diploma.



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Yojana Katti

Yojana Katti

TO
MY FATHER

An Unfathomable Pond of Inspiration

SUMMARY

This thesis presents some investigations on the thermal performance (magnitudes and efficiencies of heat retrieval) and stability aspects of three zone solar ponds round the year under local meteorological conditions of solar insolation and other factors. Sodha et al. had developed a self consistent ~~periodic~~ analysis along with computer software, for ~~studying~~ the thermal behaviour of solar ponds under two modes of heat extraction, viz. constant flow rate of heat removal fluid/
and
constant bottom temperature, for predicting the solar pond performance round the year, under local meteorological conditions.

Effect of bottom reflections in the pond has been studied by deriving a modified attenuation coefficient. The latter has been incorporated in Sodha's model to take into account the more realistic case of solar ponds with reflecting bottoms.

A comprehensive overview of the conditions for stability against convective breakdown of salt stabilized liquid columns, under large, adverse temperature gradients, has been presented. The presentation covers the most recent analysis of ^{the stability of} liquid columns ^{with large} ~~of high~~ temperature gradients backed by the experimental evidence from the UNM pond for the adequacy of the Sublayer analysis for solar ponds.

Investigations on salt diffusion in liquid columns with constant temperature gradients have also been carried out. The Soret effect which appears in the diffusion of salt in a temperature gradient has also been taken into consideration. This theory has been utilized for evaluating the predicted salt concentration profiles in solar ponds using conventional methods of salt stabilization which by constantly maintaining saturated brine at the bottom and fresh water at the top. Using the above results, stability conditions of operational three zone solar ponds have been analyzed under various thermal configurations.

An expression in the form of a polynomial has been constructed for expressing the variations of ^{Zangrando's} the stability ~~coefficients~~ ^{factors} in the operational range of temperatures and concentrations in solar ponds within an accuracy of 5%. This expression has been utilized for integrating the differential form of the stability criterion, which allows the continuous mapping of the marginal stability criterion as well as the prediction of salt concentration profiles for marginal stability in operational ponds with known temperature profiles.

PREFACE

Since the emergence of the concept of the solar pond at the beginning of this century, two spells of intensive research and development programmes - the first between 1958-1966 and the second between 1974 - 1980, have finally led to the initiation of the third stage ~~of the preparation for~~ the commercial utilization of solar pond heat in national energy strategies of many countries. With the experience gained by all round efforts in this area from fundamental research to the study of small experimental solar ponds, the present third phase activity, in fact, deals with the scaling up from the existing models of solar ponds to large sizes-of the order of 100,000 square metres and beyond, ~~and to create a base for~~ ~~forms the basis of~~ the economic commercial utilization of solar energy. Hence, comprehensive mathematical modelling of salt gradient solar ponds to predict the optimum performance and stability under varying climatological conditions, has become an important aspect. As a major example of this effort, one can cite the 'HULLPOND' and the 'COMMIX - 1A - SP' which are being used for large scale solar power plants in Israel, and U.S.A. [1].

To support an emerging national programme in India, (a country with excellent potential for exploitation of solar ponds) mathematical modelling has been taken up at the Centre of Energy Studies, Indian Institute of Technology, Delhi [2].

A mathematical model, using more elegant expressions for the attenuation coefficient of the pond fluid, the time ~~variant~~ ^{variant} ~~variation of the~~ solar insolation and other meteorological factors, has been set up. The software programs developed by this group are adequately comprehensive for predicting the thermal behaviour (temperature/heat retrieval profiles) of solar ponds for various operating conditions, viz. heat extraction at constant temperature and constant flow rates. Under these conditions, the efficiencies are predicted assuming that the solar radiation is absorbed completely at the bottom surface. The author has been working in the solar pond program at I.I.T. Delhi since 1982; the results of her investigations are presented in this thesis.

Chapter I is an overview of the comprehensive investigations leading to the initiation of efforts on the development of solar pond technology for commercial exploitation of solar energy.

In Chapter II, it has been shown that the reflected radiation from the bottom of the pond affects the magnitude of absorption of radiation in the pond fluid considerably and should be properly accounted for in the mathematical modelling of solar ponds by a suitable modification in the attenuation coefficient. Explicit relations for the modified attenuation coefficient for a single diffuse

reflection at the bottom as well as for the case of multiple reflections in the pond fluid have been obtained.

In Chapter III, prediction of the thermal behaviour of solar ponds [2] with modified subroutines introduced ^{for} ~~in~~ the attenuation coefficients, to take into account the effects of bottom reflections, has been discussed and the variation of optimum efficiency with collection temperatures and flow rates has been investigated.

A review of the mathematical theory of stabilization against convective breakdown, in double-diffusive fluid systems with heated bottoms and large temperature gradients and their applications to the stability assessment of salt stabilized solar ponds has been presented in Chapter IV.

In Chapter V, studies on "salt diffusion in fluids with non-constant temperatures" has been presented. Analytical relations have been obtained for the evaluation of salt concentrations and salt gradients in fluid layers in the non-convective zones of solar ponds by the process of natural diffusion of salt from saturated bottom layers. The nature and magnitudes of the salt concentration profiles, $C(x)$, along the depth 'x' in the thermal configurations of operational solar ponds with parabolic temperature variations along the depth (which are generally observed in practical situations) have been discussed.

Results of the investigations on the kinetics of salt diffusion in vertical liquid columns across which a constant concentration difference is maintained has been presented in Chapter VI. Analytical relations for continuous changes in the salt distribution with time, for the cases when the temperature of the column is held constant and when the temperature gradient is held constant has been developed. In the latter case, the Bessel form of the diffusion equation has been solved.

Concentration profiles, $C(x)$, at different times have been evaluated starting from the initial state with the lower half of the column at maximum concentration and the upper half at zero concentration until the near equilibrium state is reached by salt diffusion along the vertical direction. The nature and the magnitudes of salt concentration and salinity gradients as well as the time required for reaching the near equilibrium state have been discussed.

The theory has been extended to salt diffusion across stacked layers using the step function suggested by Chepurniy and Savage [56]. The decrease in time for establishing near equilibrium conditions of salt gradients in solar ponds by the stacked layer technique has been discussed.

From the results of the preceding chapters, a technique for the assessment of stability conditions in different layers along the depth has been presented in Chapter VII.

The technique involves the comparison of the observable ratio of the local temperature and salinity gradients with the stability factor of the layer ~~governed by the~~ ^{determined by the local properties of the} solution. ~~properties.~~ The application of this technique to the experimental data of the UNM pond has been discussed and shown to be in agreement with the observed stability conditions of the pond and correctly predicts the onset of convective breakdown.

The application of this technique has been further applied to solar ponds stabilized by using natural diffusion process for the establishment and maintenance of salt gradients. By the use of this technique, the suggested possibility of ^{using} lower ^{salt} concentrations ~~-(of the order of 9% (as against 25%))-~~ for ~~salt~~ stabilizing the thermal configurations of operational ponds has been discussed.

The above mentioned work has appeared partially in the form of following papers:

1. Heat Extraction from a Diffusely Reflecting Bottom Salt Gradient Solar Pond, Int. J. Solar Energy, 1 (6), 405-417 (1983).
2. Reflecting Bottom Solar Pond, Energy Conversion and Management, 24 (1), 75-88 (1984).
3. Multiple Reflection Phenomenon in Salt Gradient Solar Ponds, Solar Energy, 33 (1), 1984.

4. Kinetics of Diffusion of Salt in Solar Ponds, Int. J. Energy Research, (In Press, 1984).
5. Salt Diffusion in a Temperature Field: Application to Salinity Profiles in Solar Ponds, Int. J. Energy Research (In Press, 1984).
6. Salt Concentration and Gradient Profiles for Marginal Stability of Solar Ponds, (Communicated, 1985).
7. Stability Assessment of Salt Gradient Solar Ponds, (Communicated, 1985).
8. Temperature Dependent Salt Diffusion Characteristics of Solar Ponds, (Communicated, 1985).

In addition to these, the author has presented the following papers in conferences:

1. Thermal Behaviour of a Salt Gradient Solar Pond with Diffusely Reflecting Bottom, Proc. of National Solar Energy Convention, I.I.T. Delhi, India, 1982.
2. Effect of Diffusely Reflecting Bottom on Salt Gradient Solar Pond, Int. Solar Energy Congress, Perth, Australia, Aug., 14-19, 1983.
3. Stacked Layer Solar Pond, Proc. National Solar Energy Convention, Baroda, India, 1983.
4. Molecular Diffusion of Salt Across a Temperature Field in Solar Ponds, US-INDIA Symposium Workshop on 'Solar Energy Research & Applications', May 29-31, 1985 at University of Roorkee. (To be presented).

5. Marginal Stability Criterion for Salt Gradient Solar Ponds, Intersol 85, (To be presented).

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