

# **SCHEDULING TO MINIMIZE FLOW-TIME AND OTHER ON-LINE PROBLEMS**

by

**Muralidhara V N**

Department of Computer Science and Engineering

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*in fulfillment of the requirements of the degree of*

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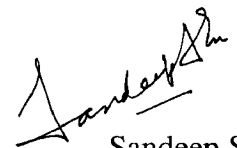
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# Certificate

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The results contained in this thesis have not been submitted to any other university or institute for the award of any degree or diploma.



Sandeep Sen

Professor

Department of Computer Science and Engineering  
Indian Institute of Technology Delhi, New Delhi 110 016



Naveen Garg

Professor

Department of Computer Science and Engineering  
Indian Institute of Technology Delhi, New Delhi 110 016

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V.N. Muralidhar  
Muralidhara V N

# Abstract

We study the problem of scheduling jobs on multiple machines in different models so as to minimize the total flow-time. We prove the following results.

1. We improve the lower bound on the approximability for the problem of minimizing flow-time on parallel machines in the off-line case from  $\Omega(\sqrt{\log P / \log \log P})$  to  $\Omega(\log^{1-\varepsilon} P)$  for any  $\varepsilon > 0$ . Here,  $P$  is the ratio of the largest to the smallest processing requirement.
2. We define a notion of  $(\alpha, \beta)$ -variability for the processing time of the jobs. We say that processing times have an  $(\alpha, \beta)$ -variability if the processing time of job  $j$  on machine  $i$  can be expressed as  $p_{ij} = a_j \cdot b_{ij} \cdot s_i$  where  $b_{ij} \in B$ ,  $|B| = \beta$  and  $1 \leq a_j \leq \alpha$ . We give an  $O(\beta \log \alpha)$  approximation for this setting in the off-line case. As special cases, we get
  - (a) an  $O(k)$  approximation when there are only  $k$  different processing times.
  - (b) an  $O(\log P)$ -approximation if each job can go on a specified subset of machines only, but has the same processing requirement on each such machine. Further, the machines can have different speeds. Here,  $P$  is the ratio of the largest to the smallest processing requirement.
3. We show that the above algorithm can be modified to get an  $O(\frac{1}{\varepsilon} \log \frac{1}{\varepsilon})$ - approximation off-line algorithm for unrelated machines if we assume that our algorithm has machines which are an  $\varepsilon$ -factor faster than the optimum algorithm's machines.

4. We give an  $O(1/\varepsilon^2)$ -approximation on-line algorithm for unrelated machines to minimize the average weighted flow-time, if we assume that our algorithm has machines which are an  $\varepsilon$ -factor faster than the optimum algorithm's machines.

Another problem we consider in this thesis is the network discovery problem. The network discovery problem involves certifying all the edges and non-edges of an unknown graph based on queries from minimal number of vertices. We will develop an on-line algorithm for approximating the optimal set cover through hitting set queries. Using this algorithm we get an  $O(\log^2 n)$ -competitive Monte Carlo randomized algorithm for the on-line version of network discovery problem.

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