A THESIS
ON HYDROMAGNETIC OSCILLATORY, BOUNDARY LAYER AND ENTRANCE REGION FLOWS

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A C K N O W L E D G E M E N T S

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C E R T I F I C A T E

This is to certify that this thesis "On Hydromagnetic Oscillatory, Boundary Layer and Entrance Region Flows", being submitted by Mr. C.V. Radhakrishna to the Indian Institute of Technology, Delhi for the award of the Degree of Doctor of Philosophy in Mathematics is a record of bonafide research work carried out by him under my supervision and guidance for the last three years and to my knowledge it has reached the standard fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or in full, to any other university or institute for the award of any degree or diploma.

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C O N T E N T S

S Y N O P S I S

Eventhough the subject of Magnetohydrodynamics has been studied by astrophysicists for many years, its importance in engineering and technology has been realized only in the last decade and a half. The earliest published works in this field are due to Hartmann and Lazarus [1937]. Recently much interest has been paid by research workers to study the response of laminar boundary layer to fluctuations in the free-stream velocity. The effects of oscillatory suction have also been investigated.

The major portion of the literature in Magnetohydrodynamics is based on the assumption of constant fluid properties. It is well known that the viscosity, electrical and thermal conductivities of many of the fluids depend markedly upon the temperature. The wide gap existing between the theoretical and experimental results cannot be bridged unless due consideration is given to the variations of these fluid properties.

Because of the numerous situations existing in aerodynamics wherein a magnetic or electric field might be used to control the drag and heat transfer on high speed vehicles, it is essential to have an accurate knowledge of the flow and heat transfer in the magnetohydrodynamic boundary-layer. The study of flow and heat transfer in channels of various cross-sections has also recieved particular attention owing to its importance in Nuclear technology and magneto-hydrodynamic power generation. The knowledge of velocity and temperature profile development is also essential from the practical design viewpoint.

The purpose of this thesis is to study various flow and heat transfer problems in hydromagnetics. The work contained in it is mainly divided into four chapters. The first chapter contains a general introduction of the problems studied, the basic equations, and a survey of the relevant literature. In the second chapter we have studied the oscillatory flow of a conducting fluid over an infinite, porous flat plate in the presence of a transverse magnetic field. The free-stream and the suction velocities are taken to be oscillating only in magnitude but not in direction with the same frequency over a mean, but with a phase difference. It is assumed that the plate is non-conducting and the conductivity of the fluid is so small as to neglect the induced magnetic field. The resulting momentum and energy equations (linear partial differential equations) have been solved by perturbation technique. Two cases have been considered; (i) thermally insulated plate

(ii) the plate maintained at a constant temperature. It is observed that the effect of the magnetic field is to prevent the reverse flow which occurs near the plate at higher frequencies and amplitudes of the free stream velocity fluctuations. In contrast to the case of constant suction velocity, the wall temperature and heat flux are observed to be independent of the frequency.

In chapter III we have considered the steady flow and heat transfer inside the boundary layer of a conducting fluid over a porous (suction velocity varying according to the law $v \propto \frac{v_o}{\sqrt{x}}$) semi-infinite flat plate in the presence of a transverse magnetic field. The chapter is divided into two sections. In the first section we have considered the case when the magnetic field is varying according to the law $H \propto \frac{H_o}{\sqrt{x}}$. This admits of similarity solutions and the resultant momentum and energy equations reduce to a system of ordinary differential equations. In the second section the magnetic field is taken to be constant. In this case the partial differential equations governing the flow are analyzed by series expansion technique. The viscosity, electrical and thermal conductivities of the fluid are taken to be linearly dependent upon temperature. The form of the electrical conductivity is chosen in such a way that far from the plate its value becomes zero, because in some of the practical problems of aerodynamics the fluid near a surface is ionized and, therefore, electrically conducting whereas farther off the surface it remains unaffected. The consideration of the effects of variable fluid properties makes the problem highly non-linear. This non-linear boundary value problem is solved numerically for the stream function ψ and temperature θ by the fourth order Runge-Kutta process as out lined by Gill. Certain initial values for some of the dependent variables have to be assumed so as to start the integration and these are corrected to a desired degree of accuracy by a self iterative correction procedure.

It is observed that with the increase in the dependence of viscosity on temperature, the skin-friction decreases and the Nusselt number increases. The increasing dependence of the thermal conductivity on temperature increases the Nusselt number whereas no appreciable change is observed in the skin-friction. An increase in the suction velocity is followed by an increase in both the skin-friction and Nusselt number. Increasing dependence of electrical conductivity on temperature decreases both the skin-friction and Nusselt number.

Chapter IV is devoted to the study of hydro-magnetic channel flows. Here we have considered the flow of a conducting fluid between two parallel plates in the presence of a transverse magnetic field. The chapter is divided into three sections. In the first section the problem of fully developed flow under the assumption of weak dependence of fluid properties upon temperature is investigated by perturbation technique. In contrast to the case of constant fluid properties it is observed that the electric field effects the velocity profile. In the second section, the finite difference method [originally developed by Hartree (1949) to solve the boundary layer equation] which has been adopted by Ram Manohar [1965,66] to study the development of velocity profiles in channel flows has been improved inview of computational time and accuracy to study the velocity and temperature profile development in the entrance region under the assumption of constant fluid properties. In the third section the same problem has been solved by taking the effects ofvariations in the fluid properties into account. It is observed that with the increasing dependence of viscosity and electrical conductivity on temperature is followed by an increase in the development length whereas with the increasing dependence of thermal conductivity on temperature is followed by a decrease in the development length.