

**ROBUST CONTROLLER DESIGN AND STABILITY  
ANALYSIS OF TWO-INPUT DC-DC CONVERTERS**

**B. AMARENDRA REDDY**



**DEPARTMENT OF ELECTRICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY DELHI**

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**by**

**B. AMARENDRA REDDY**

**Department of Electrical Engineering**

**Submitted**

**in fulfillment of the requirements of the degree of Doctor of Philosophy**

**to the**



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**To Lord Almighty**

**To My Parents**

**Sri. B. Vema Reddy (late), Smt. B.Padmavathi**

**To My Wife and Son**

**Smt. B. Jaya Prada and B. Gireesh Reddy**

**To My Sister**

**Smt. B. Chandrakala and Family**

**To My Teachers**

## CERTIFICATE

This is to certify that the thesis entitled “**Robust Controller Design and Stability Analysis of Two-input DC-DC Converters**” being submitted by **Mr. B. Amarendra Reddy** for the award of degree of **Doctor of Philosophy** is a record of a bonafide research work carried out by him in the Department of Electrical Engineering of Indian Institute of Technology Delhi, New Delhi. Mr. B. Amarendra Reddy worked under my guidance and supervision and has fulfilled the requirement for the submission of the thesis, which to my knowledge has reached the requisite standards. The matter embodied in this thesis has not been submitted to any other University or Institute for the award of any Degree or Diploma.

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Dr. M. Veerachary  
Professor  
Department of Electrical Engineering  
Indian Institute of Technology Delhi  
New Delhi-110016

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## ABSTRACT

In this thesis, power management aspects of non-isolated two-input integrated dc-dc converters are investigated. The two integrated converters considered for the investigation are: (i) two-input fourth-order integrated converter, and (ii) two-input sixth-order integrated converter. From voltage conversion point of view, both these converters belongs to a class of buck or buck-boost type of conversion. Both these topologies are capable of drawing power from two different input dc-sources and feeding to a common load. Though these integrated converters are able to achieve desired voltage conversion ratios along with feasible load power management with reduced number components, but stability and robustness related issues are more prominent (i.e. control-loops are more than one) if multi-loop control strategies are adopted in the load management. These issues primarily arise due to the integrated structure of the converter and transfer of power to load which is drawn from multiple sources has to flow through common energy storage elements. Since some portion of the integrated converter is common to both sources, the dynamical interactions arise due to the rate at which power is drawn from different sources, and the rate at which energy is stored and released in various sections of the circuit by the inductive and capacitive elements. Hence, design and development of suitable multi-variable control strategies for two-input integrated dc-dc converters are investigated in this thesis.

In order to address the stability and robustness issues in the two-input integrated converters as discussed above, firstly the cause of interactions is identified and thereafter interaction quantification methods are adopted to explore the degree of interactions among the control-loops and their quantification. On the basis of this quantification, pairing between the controlling input and the controlled output variables is formulated. Further, this study also revealed that for the integrated converter systems under investigation, two control-loops are sufficient enough to

ensure feasible power management which are: load voltage regulation loop and source current regulation loop.

Two different control strategies are investigated in this thesis to achieve reliable power conversion with two-input integrated converters, which are: (i) decentralized control strategy with two diagonal controllers, and (ii) centralized control strategy with full-order controller. For the decentralized controller design, firstly an effective transfer function method is extended. In the process of design, it has been identified that through this method, the final resulting controller accuracy depends on the plant interactions and how effectively they are included in the diagonal non-interactive transfer functions. To overcome these limitations, stability boundary locus based stabilizing regions are generated for the decentralized controller parameters. Though this methodology is a bit complex but the final design guarantees feasible controller parameter region which will ensure the closed-loop system with desired stability/ robustness range.

Though the decentralized control strategy is simple to implement but the control-loop dynamic performance may not be sufficient and it depends on the operating point dependent plant interactions. In an attempt to enhance the control-loop performance and to achieve feasible power management of integrated converters, an  $H_\infty$  loopshaping based centralized (full-order) controller is designed. This methodology is a combination of the classical loopshaping along with  $H_\infty$  robust stabilization and hence it is possible to ensure both stability and robustness of the integrated converter systems.

Both the controllers introduced above (decentralized and centralized controller) ensures the integrated converters stabilization but exhibits differences in terms of performance due to the presence of interactions within the integrated converters. To ascertain these performance aspects of integrated converters, the stability and robustness issues are addressed in this thesis by

introducing a singular value analysis. The  $\mu$ -plots of integrated converters for different operating conditions pertaining to nominal performance, robust stability and robust performance are generated for both the decentralized and centralized controllers. For all these cases, the ISE performance indices are also computed. These singular value analysis investigations revealed that the TIFOI converter along with centralized converter yields not only the robust performance feature but also exhibits better dynamic performance. The decentralized and centralized controllers effectiveness is demonstrated through simulations. Sample experimental results are presented to validate the design and analysis.

The main contributions of this thesis are: (i) To ensure reliable power management in the integrated dc-dc converter by using an appropriate control strategy, an extensive mathematical analysis is established by means of which pairing and control-loop interactions prevailing within the integrated converters are identified, (ii) many hidden issues prevailing in the integrated converters are identified using the multi-variable control theory which arise due to the presence of common energy storage elements. With the aim to avoid the complexities and yet establish a reliable control strategy, firstly a decentralized controller design is formulated using the second-order plus time-delay equivalent transfer functions. Though this methodology is simple but needs iterations before converging into the final control strategy. To avoid this exercise, stabilizing regions are generated based on stability boundary locus method for the controller parameters, (iii) Though these decentralized controllers are able to ensure the task of power management, due to the control-loop interactions the dynamic performance may be affected. In an attempt to enhance the dynamic performance, a centralized control strategy is evolved, and (iv) The effectiveness of the decentralized and centralized controllers for the integrated dc-dc converters is rigorously investigated through singular value analysis. Nominal performance, robust stability

and robust performance depicting  $\mu$ -plots are generated to ascertain the impact of decentralized and centralized controllers. Further, the integral square error (ISE) performance indices are computed for both which reveal that the centralized controller result in lower ISE performance index than the decentralized one and hence the centralized controller exhibits better dynamic performance.

## सार

इस थीसिस में, गैर-पृथक दो-इनपुट एकीकृत डीसी-डीसी कन्वर्टर के बिजली प्रबंधन पहलुओं की जांच की जाती है। जांच के लिए जिन दो एकीकृत कन्वर्टर पर विचार किया गया है, वे हैं: (i) दो-इनपुट चौथे क्रम के एकीकृत कनवर्टर, और (ii) दो-इनपुट छठे क्रम के एकीकृत कनवर्टर। वोल्टेज रूपांतरण बिंदु से, ये दोनों कन्वर्टर हिरन या हिरन-बूस्ट प्रकार के रूपांतरण के एक वर्ग से संबंधित हैं। ये दोनों टोपोलॉजी दो अलग-अलग इनपुट डीसी-स्रोतों से बिजली खींचने और एक सामान्य भार को खिलाने में सक्षम हैं। हालांकि ये एकीकृत कन्वर्टर कम संख्या के घटकों के साथ संभव लोड बिजली प्रबंधन के साथ वांछित वोल्टेज रूपांतरण अनुपात को प्राप्त करने में सक्षम हैं, लेकिन मल्टी-लूप नियंत्रण रणनीतियों को अपनाया जाने पर स्थिरता और मजबूती से संबंधित मुद्दे अधिक प्रमुख हैं (यानी नियंत्रण-छोर एक से अधिक हैं) भार प्रबंधन में। ये मुद्दे मुख्य रूप से कनवर्टर की एकीकृत संरचना और लोड करने के लिए बिजली के हस्तांतरण के कारण उत्पन्न होते हैं, जो कई स्रोतों से खींचा जाता है और आम भंडारण भंडारण तत्वों के माध्यम से प्रवाह होता है। चूंकि एकीकृत कनवर्टर का कुछ हिस्सा दोनों स्रोतों के लिए सामान्य है, इसलिए विभिन्न स्रोतों से जिस दर पर बिजली खींची जाती है, उस दर के कारण गतिशील अंतःक्रियाएं उत्पन्न होती हैं, और जिस दर पर ऊर्जा को संग्रहीत किया जाता है और प्रेरक द्वारा सर्किट के विभिन्न वर्गों में जारी किया जाता है। कैपेसिटिव तत्व। इसलिए, इस थीसिस में दो-इनपुट एकीकृत डीसी-डीसी कन्वर्टर के लिए उपयुक्त बहु-चर नियंत्रण रणनीतियों के डिजाइन और विकास की जांच की जाती है।

ऊपर चर्चा किए गए अनुसार दो-इनपुट एकीकृत कन्वर्टर में स्थिरता और मजबूती के मुद्दों को संबोधित करने के लिए, सबसे पहले बातचीत के कारण की पहचान की जाती है और उसके बाद नियंत्रण-छोरों और उनके परिमाण के बीच बातचीत की डिग्री का पता लगाने के लिए बातचीत मात्रा का ठहराव के तरीकों को अपनाया जाता है। इस परिमाण के आधार पर, नियंत्रित इनपुट और नियंत्रित आउटपुट चर के बीच युग्मन तैयार किया जाता है। इसके अलावा, इस अध्ययन से यह भी पता चला है कि जांच के तहत

एकीकृत कन्वर्टर सिस्टम के लिए, दो नियंत्रण-लूप पर्याप्त हैं जो व्यवहार्य बिजली प्रबंधन सुनिश्चित करने के लिए पर्याप्त हैं: लोड वोल्टेज विनियमन लूप और स्रोत वर्तमान विनियमन लूप।

इस इनपुट में दो अलग-अलग नियंत्रण रणनीतियों की जांच दो-इनपुट एकीकृत कन्वर्टर्स के साथ विश्वसनीय बिजली रूपांतरण प्राप्त करने के लिए की जाती है, जो हैं: (i) दो विकर्ण नियंत्रकों के साथ विकेंद्रीकृत नियंत्रण रणनीति, और (ii) पूर्ण-आदेश नियंत्रक के साथ केंद्रीकृत नियंत्रण रणनीति। विकेंद्रीकृत नियंत्रक डिजाइन के लिए, सबसे पहले एक प्रभावी हस्तांतरण फ़ंक्शन विधि विस्तारित की जाती है। डिजाइन की प्रक्रिया में, यह पता चला है कि इस पद्धति के माध्यम से, अंतिम परिणामी नियंत्रक सटीकता संयंत्र की बातचीत पर निर्भर करती है और वे कितनी प्रभावी रूप से विकर्ण गैर-संवादात्मक हस्तांतरण कार्यों में शामिल हैं। इन सीमाओं को पार करने के लिए, विकेंद्रीकृत नियंत्रक मापदंडों के लिए स्थिरता सीमा क्षेत्र आधारित स्थिरीकरण क्षेत्र उत्पन्न होते हैं। हालांकि यह कार्यप्रणाली थोड़ी जटिल है लेकिन अंतिम डिजाइन संभव नियंत्रक पैरामीटर क्षेत्र की गारंटी देता है जो वांछित स्थिरता / मजबूती सीमा के साथ बंद लूप सिस्टम को सुनिश्चित करेगा।

हालांकि विकेंद्रीकृत नियंत्रण रणनीति को लागू करना सरल है, लेकिन नियंत्रण-लूप गतिशील प्रदर्शन पर्याप्त नहीं हो सकता है और यह ऑपरेटिंग बिंदु निर्भर संयंत्र बातचीत पर निर्भर करता है। नियंत्रण-लूप प्रदर्शन को बढ़ाने और एकीकृत कन्वर्टर्स के व्यवहार्य शक्ति प्रबंधन को प्राप्त करने के प्रयास में, एक लूपशाॅप आधारित केंद्रीकृत (पूर्ण-क्रम) नियंत्रक को डिजाइन किया गया है। यह पद्धति मजबूत स्थिरीकरण के साथ-साथ शास्त्रीय लूपशैपिंग का एक संयोजन है और इसलिए यह एकीकृत कन्वर्टर सिस्टम की स्थिरता और मजबूती दोनों सुनिश्चित करना संभव है।

ऊपर दिए गए दोनों नियंत्रक (विकेंद्रीकृत और केंद्रीकृत नियंत्रक) एकीकृत कन्वर्टर्स स्थिरीकरण सुनिश्चित करते हैं, लेकिन एकीकृत कन्वर्टर्स के भीतर इंटरैक्शन की उपस्थिति के कारण प्रदर्शन के मामले में अंतर प्रदर्शित करता है। एकीकृत कन्वर्टर्स के इन प्रदर्शन पहलुओं का पता लगाने के लिए, इस

थीसिस में एक विलक्षण मूल्य विश्लेषण शुरू करके स्थिरता और मजबूती के मुद्दों को संबोधित किया जाता है। नाममात्र प्रदर्शन, मजबूत स्थिरता और मजबूत प्रदर्शन से संबंधित विभिन्न परिचालन स्थितियों के लिए एकीकृत कन्वर्टर्स के-प्लॉट विकेंद्रीकृत और केंद्रीकृत नियंत्रकों दोनों के लिए उत्पन्न होते हैं। इन सभी मामलों के लिए, आईएसई प्रदर्शन सूचकांक की गणना भी की जाती है। इन विलक्षण मूल्य विश्लेषण जांच से पता चला कि TIFOI कनवर्टर केंद्रीकृत कनवर्टर पैदावार के साथ न केवल मजबूत प्रदर्शन सुविधा देता है, बल्कि बेहतर गतिशील प्रदर्शन भी दिखाता है। सिमुलेशन के माध्यम से विकेंद्रीकृत और केंद्रीकृत नियंत्रकों की प्रभावशीलता का प्रदर्शन किया जाता है। नमूना प्रयोगात्मक परिणाम डिजाइन और विश्लेषण को मान्य करने के लिए प्रस्तुत किए जाते हैं।

इस थीसिस के मुख्य योगदान हैं: (i) एक उचित नियंत्रण रणनीति का उपयोग करके एकीकृत डीसी-डीसी कनवर्टर में विश्वसनीय बिजली प्रबंधन सुनिश्चित करने के लिए, एक व्यापक गणितीय विश्लेषण की स्थापना की जाती है जिसके माध्यम से जोड़ी और नियंत्रण-लूप इंटरैक्शन एकीकृत के भीतर प्रचलित होते हैं कन्वर्टर्स की पहचान की जाती है, (ii) एकीकृत कन्वर्टर्स में प्रचलित कई छिपे हुए मुद्दों को बहु-चर नियंत्रण सिद्धांत का उपयोग करके पहचाना जाता है जो आम ऊर्जा भंडारण तत्वों की उपस्थिति के कारण उत्पन्न होते हैं। जटिलताओं से बचने और अभी तक एक विश्वसनीय नियंत्रण रणनीति स्थापित करने के उद्देश्य से, सबसे पहले एक विकेंद्रीकृत नियंत्रक डिजाइन को दूसरे क्रम के साथ-साथ समय-विलंब समकक्ष हस्तांतरण कार्यों का उपयोग करके तैयार किया गया है। हालांकि यह कार्यप्रणाली सरल है, लेकिन अंतिम नियंत्रण रणनीति में परिवर्तित करने से पहले पुनरावृत्तियों की आवश्यकता है। इस अभ्यास से बचने के लिए, नियंत्रक मापदंडों के लिए स्थिरता सीमा नियंत्रण रेखा के आधार पर स्थिर क्षेत्रों को उत्पन्न किया जाता है, (iii) हालांकि ये विकेंद्रीकृत नियंत्रक पावर प्रबंधन के कार्य को सुनिश्चित करने में सक्षम हैं, क्योंकि नियंत्रण-लूप इंटरैक्शन के कारण गतिशील प्रदर्शन प्रभावित हो सकता है। गतिशील प्रदर्शन को बढ़ाने के प्रयास में, एक केंद्रीकृत नियंत्रण रणनीति विकसित की जाती है, और (iv)

एकीकृत डीसी-डीसी कन्वर्टर के लिए विकेंद्रीकृत और केंद्रीकृत नियंत्रकों की प्रभावशीलता को एकवचन मूल्य विश्लेषण के माध्यम से कड़ाई से जांच की जाती है। विकेंद्रीकृत और केंद्रीकृत नियंत्रकों के प्रभाव का पता लगाने के लिए नाममात्र प्रदर्शन, मजबूत स्थिरता और भूखंडों को दर्शाने वाला मजबूत प्रदर्शन उत्पन्न होता है। इसके अलावा, अभिन्न वर्ग त्रुटि (ISE) प्रदर्शन सूचकांक दोनों के लिए गणना की जाती है, जो यह बताता है कि विकेंद्रीकृत की तुलना में कम ISE प्रदर्शन सूचकांक में केंद्रीकृत नियंत्रक परिणाम और इसलिए केंद्रीकृत नियंत्रक बेहतर गतिशील प्रदर्शन प्रदर्शित करता है।

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## LIST OF SYMBOLS

Symbol	Description
$f_s$	Switching frequency
$T_s$	Time period
$d$	Duty-ratio
$t$	Instantaneous time instant
$S_1$	Switch $S_1$
$S_2$	Switch $S_2$
$D_1$	Diode $D_1$
$D_2$	Diode $D_2$
$V_{g1}$	Source-1 voltage
$V_{g2}$	Source-2 voltage
$\hat{i}_{g1}$	Small-signal variation in source-1 current
$\hat{i}_{g2}$	Small-signal variation in source-2 current
$L_1$	Inductor $L_1$
$L_2$	Inductor $L_2$
$L_3$	Inductor $L_3$
$C_1$	Capacitor $C_1$
$C_2$	Capacitor $C_2$
$C_3$	Capacitor $C_3$
$i_1$	Current through inductor $L_1$
$i_2$	Current through inductor $L_2$
$i_3$	Current through inductor $L_3$
$i_{c1}$	Current through capacitor $C_1$
$i_{c2}$	Current through capacitor $C_2$
$i_{c3}$	Current through capacitor $C_3$

$r_1$	ESL of inductor $L_1$
$r_2$	ESR of inductor $L_2$
$r_3$	ESR of inductor $L_3$
$r_{c1}$	ESR of capacitor $C_1$
$r_{c2}$	ESR of capacitor $C_2$
$r_{c3}$	ESR of capacitor $C_3$
$v_{L1}$	Voltage across inductor $L_1$
$v_{L2}$	Voltage across inductor $L_2$
$v_{L3}$	Voltage across inductor $L_3$
$v_{c1}$	Voltage across capacitor $C_1$
$v_{c2}$	Voltage across capacitor $C_2$
$v_{c3}$	Voltage across capacitor $C_3$
$\Delta i_1$	Ripple in inductor $L_1$ current
$\Delta i_2$	Ripple in inductor $L_2$ current
$\Delta i_3$	Ripple in inductor $L_3$ current
$\Delta v_{c1}$	Ripple in capacitor $C_1$ voltage
$\Delta v_{c2}$	Ripple in capacitor $C_2$ voltage
$\Delta v_{c3}$	Ripple in capacitor $C_3$ voltage
$R$	Load resistance
$I_o$	Load current
$v_o$	Load voltage
$A$	System matrix
$B$	Input matrix
$E$	Output matrix
$F$	Feed-through matrix
$u$	Input/control vector
$x$	State vector
$y$	Output

$\hat{x}$	Small-signal variation in input
$\hat{y}$	Small-signal variation in output
$\hat{v}_o$	Small-signal variation in load voltage
W	Watts
$P_o$	Power output
$I_{dent}$	Identity matrix
$\gamma_1$	Gamma-1
$\gamma_2$	Gamma-2
s	Laplace domain operator
z	Discrete domain operator
$G_{11}$	Control-1-to-output $v_o$
$G_{12}$	Control-2-to-output $v_o$
$G_{21}$	Control-2-to-output $i_{g2}$
$G_{22}$	Control-2-to-output $i_{g2}$
$\hat{d}_1$	Duty-ratio-1 perturbation
$\hat{d}_2$	Duty-ratio-2 perturbation
$nT_s$	Sampling time instant
$t_d$	Time delay
$\phi$	Phi
adj	Adjoint
$N_c$	Numerator matrix
$\Delta_c$	Denominator