

**ADAPTIVE IDENTIFICATION AND CONTROL OF
SWITCHED LINEAR SYSTEMS: A MEMORY
AUGMENTATION APPROACH**

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DEPARTMENT OF ELECTRICAL ENGINEERING

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**ADAPTIVE IDENTIFICATION AND CONTROL OF
SWITCHED LINEAR SYSTEMS: A MEMORY
AUGMENTATION APPROACH**

by

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Submitted

in fulfillment of the requirements of the degree of Doctor of Philosophy

to the



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DEDICATED TO

I dedicate this thesis to my mentors, my parents, my brother, my wife, every person who helped me directly or indirectly, and the Almighty without whom none of my success would be possible.

CERTIFICATE

This is to certify that the thesis entitled **Adaptive Identification and Control of Switched Linear Systems: A Memory Augmentation Approach** submitted by **Pritesh Patel(Entry No. 2018EEZ8135)** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the bona fide research work carried out by him under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree. The results contained in this thesis have not been submitted either in part or in full to any other University or Institute for the award of any degree or diploma to the best of my knowledge.

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Pritesh Patel
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ABSTRACT

This dissertation undertakes a comprehensive exploration of the [Switched Model Reference Adaptive Control \(S-MRAC\)](#) strategies for parameter identification and control of switched [Multi Input Multi Output \(MIMO\)](#) linear systems and adaptive fault-tolerant methodologies applied to [Euler-Lagrange \(EL\)](#) systems. The primary objective is to enhance parameter convergence, tracking performance, and [Fault Tolerant Control \(FTC\)](#) for switched dynamical systems.

The investigation delves into the analysis of memory augmentation and the [Intermittently Initially Exciting \(IIE\)](#) condition for switched linear systems. The study extends to compensating multiple actuator faults in [EL](#) systems through adaptive control techniques. The incorporation of memory augmentation and [IIE](#) condition proves instrumental in establishing globally exponential stability of overall error which includes tracking error and parameter estimation errors of all the subsystems for switched linear systems, a result not previously found in the literature. The absence of memory augmentation and/or non-satisfaction of the [IIE](#) condition limits stability outcomes to either Lyapunov stability or asymptotic stability. The [IIE](#) condition, a more generalized form of the [Initially Exciting \(IE\)](#) condition, is demonstrated to be less restrictive than the [Persistently Exciting \(PE\)](#) condition for ensuring parameter convergence in adaptive systems. Memory augmentation also facilitates parameter learning during the inactive phase of subsystems, thereby enhancing overall tracking performance. A novel tunable and unified dwell time expression is derived, proving beneficial even when the reference signal is not sufficiently exciting.

The dissertation sequentially examines online adaptive identification, focusing on the accurate identification of switched system parameters. Subsequently, [S-MRAC](#) is proposed for a common reference model, and stability is analyzed using a common Lyapunov function. The analysis extends to a more general scenario wherein the reference model is also switched, employing multiple Lyapunov functions to assess the stability of overall error dynamics. The dissertation concludes with a dedicated exploration of adaptive actuator fault-tolerant control for the [EL](#) system, wherein single and multiple faults are compensated using adaptive control methods. Well-known projection operation in adaptive system literature is shown to compensate for the infinite number of actuator faults. Collectively, these contributions advance the field of adaptive control and fault-tolerant methodologies, offering a unified framework for the design and simulation of control systems in practical applications.

सार

यह शोध प्रबंध स्विचड मल्टीपल-इनपुट मल्टीपल-आउटपुट (एमआईएमओ) लीनियर सिस्टम और इलेक्ट्रोमैके-निकल (ईएल) सिस्टम पर लागू अनुकूली दोष-सहिष्णु पद्धतियों के पैरामीटर पहचान और नियंत्रण के लिए मॉडल संदर्भ अनुकूली नियंत्रण रणनीतियों की व्यापक खोज करता है। प्राथमिक उद्देश्य स्विचड डायनेमिक सिस्टम के लिए पैरामीटर अभिसरण, ट्रैकिंग प्रदर्शन और दोष-सहिष्णु नियंत्रण को बढ़ाना है।

यह जांच स्विचड एलटीआई सिस्टम के लिए मेमोरी वृद्धि और इंटीग्रल इनपुट-टू-एरर (आईआईई) स्थिति के विश्लेषण पर प्रकाश डालती है। यह अध्ययन अनुकूली नियंत्रण तकनीकों के माध्यम से ईएल सिस्टम में कई एक्चुएटर दोषों की भरपाई करने तक फैला हुआ है। मेमोरी वृद्धि और आईआईई स्थिति का समावेश समग्र त्रुटि की समान रूप से विश्व स्तर पर घातीय स्थिरता स्थापित करने में महत्वपूर्ण साबित होता है जिसमें स्विच किए गए एलटीआई सिस्टम के लिए सभी उपप्रणालियों की ट्रैकिंग त्रुटि और पैरामीटर अनुमान त्रुटियां शामिल होती हैं, जिसका परिणाम पहले साहित्य में नहीं मिला था। स्मृति वृद्धि की अनुपस्थिति और/या आई आई ई स्थिति की गैर-संतुष्टि स्थिरता परिणामों को ल्यपुनोव स्थिरता या स्पर्शोन्मुख स्थिरता तक सीमित कर देती है। आईआईई स्थिति, इंटीग्रल इनपुट-टू-स्टेट (आईआईएस) स्थिति का अधिक सामान्यीकृत रूप, अनुकूली प्रणालियों में पैरामीटर अभिसरण सुनिश्चित करने के लिए लगातार उत्तेजना (पीई) स्थिति की तुलना में कम प्रतिबंधात्मक साबित होती है। मेमोरी वृद्धि उप-प्रणालियों के निष्क्रिय चरण के दौरान पैरामीटर सीखने की सुविधा भी प्रदान करती है, जिससे समग्र ट्रैकिंग प्रदर्शन में वृद्धि होती है। एक नवीन ट्यून करने योग्य और एकीकृत ड्वेल टाइम अभिव्यक्ति प्राप्त की गई है, जो तब भी फायदेमंद साबित होती है जब संदर्भ संकेत पर्याप्त रूप से रोमांचक नहीं होता है।

शोध प्रबंध क्रमिक रूप से ऑनलाइन अनुकूली पहचान की जांच करता है, जो स्विच किए गए सिस्टम मापदंडों की सटीक पहचान पर ध्यान केंद्रित करता है। इसके बाद, एक सामान्य संदर्भ मॉडल के लिए स्विचिंग मॉडल संदर्भ अनुकूली नियंत्रण (एस-एमआरएसी) प्रस्तावित किया गया है, और एक सामान्य ल्यपुनोव फ्रंक्शन का उपयोग करके स्थिरता का विश्लेषण किया जाता है। विश्लेषण अधिक सामान्य परिदृश्य तक फैला हुआ है जिसमें संदर्भ मॉडल को भी स्विच किया जाता है, समग्र त्रुटि गतिशीलता की स्थिरता का आकलन करने के लिए कई ल्यपुनोव कार्यों को नियोजित किया जाता है। शोध प्रबंध ईएल प्रणाली के लिए अनुकूली एक्चुएटर दोष-सहिष्णु नियंत्रण की एक समर्पित खोज के साथ समाप्त होता है, जिसमें अनुकूली नियंत्रण विधियों का उपयोग करके एकल और एकाधिक दोषों की भरपाई की जाती है। अनुकूली प्रणाली साहित्य में प्रसिद्ध प्रक्षेपण ऑपरेशन को अनंत संख्या में एक्चुएटर दोषों की भरपाई के लिए दिखाया गया है। सामूहिक रूप से, ये योगदान व्यावहारिक अनुप्रयोगों में नियंत्रण प्रणालियों के डिजाइन और कार्यान्वयन के लिए एक एकीकृत ढांचे की पेशकश करते हुए, अनुकूली नियंत्रण और दोष-सहिष्णु पद्धतियों के क्षेत्र को आगे बढ़ाते हैं।

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Acronyms

EL Euler-Lagrange. [iii](#), [8](#), [10](#), [11](#), [106](#)

FTC Fault Tolerant Control. [iii](#), [16](#)

GES Globally Exponentially Stable. [9](#), [10](#), [20](#), [21](#), [31](#), [33](#), [37](#), [38](#), [57–59](#), [64](#), [67](#), [71](#), [82](#), [83](#), [103](#)

GUB Globally Ultimately Bounded. [37](#), [50](#), [55](#), [56](#), [79](#), [82](#)

IE Initially Exciting. [iii](#), [9](#), [17](#)

IIE Intermittently Initially Exciting. [iii](#), [9](#), [10](#), [20](#), [21](#), [26](#), [28–32](#), [35](#), [37](#), [38](#), [48–51](#), [53](#), [54](#), [57](#), [59–61](#), [63](#), [66–68](#), [76](#), [78](#), [80](#), [82](#), [86](#), [103](#)

MIMO Multi Input Multi Output. [iii](#), [35](#), [38](#)

MRAC Model Reference Adaptive Control. [5](#), [6](#), [58](#)

PE Persistently Exciting. [iii](#), [6](#), [8–10](#), [17](#), [37](#), [38](#), [46](#), [59](#), [60](#), [86](#)

PLOE Partial Loss of Effectiveness. [7](#), [9](#), [87–89](#), [92](#), [104](#), [107](#)

S-MRAC Switched Model Reference Adaptive Control. [iii](#), [5](#), [10](#), [36](#), [44](#)

Nomenclature

\mathbf{R} Real Space

\mathbf{R}^n Real Vector Space of Dimension n

$\mathbf{R}^{n \times m}$ Real Matrix Space of Dimension $n \times m$

$\mathbf{I}_{n \times n}$ Identity Matrix of Dimension $n \times n$

$\lambda_{\min}(\mathbf{P})$ Minimum Eigen Value of Matrix \mathbf{P}

$\lambda_{\max}(\mathbf{P})$ Maximum Eigen Value of Matrix \mathbf{P}

\otimes Kronecker Product

$vec(\mathbf{Z})$ Vector achieved by stacking the columns of matrix \mathbf{Z}

$tr\{\bullet\}$ Trace of a Matrix

$\mathbf{0}_{n \times m}$ Matrix sized $n \times m$ with all elements set to zero

\mathbf{N}^+ Set of all positive natural numbers

$\|\mathbf{x}\|$ Euclidean Norm of a Vector \mathbf{x}

\exists There Exists

\forall For All

$\|\mathbf{x}\|_1$ 1-Norm of a Vector \mathbf{x}

$\|\mathbf{A}\|_F$ Frobenius Norm of a Matrix \mathbf{A}