

A NUMERICAL STUDY OF TURBULENT CHANNEL
FLOW UNDER HIGH TEMPERATURE GRADIENT

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DEPARTMENT OF APPLIED MECHANICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI.

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A NUMERICAL STUDY OF TURBULENT CHANNEL FLOW UNDER HIGH TEMPERATURE GRADIENT

A THESIS

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of

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THESIS CERTIFICATE

This is to certify that the thesis titled **A NUMERICAL STUDY OF TURBULENT CHANNEL FLOW UNDER HIGH TEMPERATURE GRADIENT**, submitted by **Syed Mohd Yahya**, to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bona fide record of the research work done by him under our supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

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ABSTRACT

Thermal Large Eddy Simulation (TLES) is performed to study the low Mach number turbulent channel flow with variable thermofluid properties in forced and mixed convection environment. In this flow configuration top wall (hot) is maintained at a higher temperature than the bottom wall (cold). The temperature ratio, $R_\theta = T_{hot}/T_{cold}$ is used to quantify the heating effect and varied from $R_\theta=1$ to 5 for forced convective case and $R_\theta=2$ & 3 for mixed convection case. All simulations are performed at a constant Prandtl number $Pr=0.71$ and at various shear Reynolds numbers.

Firstly, an extensive study of various subgrid models is carried out to ascertain the suitability of the most optimum model for the said problem. The results for two dynamic models namely dynamic Smogorinsky model and WALE (Wall Adaptive Local Eddy viscosity) model are presented for Reynolds stresses. For thermal part the subgrid scale diffusivity model with constant subgrid scale Prandtl number and its dynamic formulation is used. It is concluded that WALE and dynamic thermal model give best optimum results for dynamic and thermal part respectively.

In the case of forced convective turbulence in a bi-periodic channel more emphasis is put to address the effects of properties variation. The results indicate that the average and turbulent fields undergo remarkable changes as the Reynolds number is varied over a broad range. Compared to the case of isothermal flow with constant viscosity, it is observed that turbulence is enhanced in the cold side of the channel, characterized by locally lower viscosity whereas a decrease of turbulent kinetic energy is found near the hot wall. The turbulent structures via H criteria of high vorticity shows very short and densely populated vortices near the cold wall whereas long streaky structures or large elongated vortices are observed at the hot wall. Q invariant almost eradicates all the streaky structures near the hot wall as a consequence of re-laminarization. Spectral study reveals that turbulence is completely suppressed at the hot side

of the channel at a large temperature ratio because no inertial zone is obtained (i.e. index of Kolmogorov scaling law is zero) from the spectra in these region. In order to have more insight in the physics of the problem dynamical behavior of the coherent structures in a turbulent channel flow is investigated via proper orthogonal decomposition (POD). The POD technique is applied to the fluctuating part of the velocity to study the temporal evolution of the most energetic modes. It is observed that dominant flow structure are elongated in streamwise direction which are further distorted due to the interaction with the bean shaped propagating modes. Temperature stratification affects the interaction process during turbulent burst event resulting in a chugging or relaminarizing phenomenon. Furthermore temporal evolution of energy and entropy is presented to capture the consistent pattern of growth and decay. It is observed that thermal stratification decreases the bursting rate by slowing the breaking mechanism of streamwise elongated modes to larger number of bean shaped modes which results in relaminarization of the flow near the hot wall.

Large eddy simulations of stably stratified, turbulent channel flow subjected to a large temperature gradient have been performed by considering a broad spectrum of stratification. Due to a large thermal gradient across the channel, the temperature-dependent fluid properties like viscosity (μ), density (ρ), and thermal conductivity (κ) are considered as variables. With increased stratification, a cluster of laminar patches appears in the near-wall region, and turbulent momentum and buoyancy fluxes are suppressed drastically in the core of the channel due to the formation of internal gravity waves. Variable viscosity results in flow relaminarization on the hot side of the channel (where viscosity is higher). A mechanistic model for wall heat transfer is developed for buoyancy-affected flows. Qualitatively and quantitatively the pronounced modifications in turbulent structures and flow statistics are observed. In a stably stratified turbulent flow, the computations based on Boussinesq assumption (uniform fluid properties) for strongly temperature dependent fluid properties like viscosity and thermal conductivity results in an inaccurate estimation of momentum and heat transfer coefficients. Therefore a clear demonstration about the specific role of thermo-fluid properties and their deviation under non-Boussinesq condition with that of Boussinesq case is performed. Results for different ($Ri_\tau = Gr/Re_\tau^2$) under a fixed temperature boundary conditions by keeping Grashof number (Gr) constant and

varying shear Reynolds number (Re_τ) are presented. It is found that as the Richardson number increases, the effective thermal convection and eventually the Nusselt number is reduced. Viscosity smears the buoyant thermal plume generation resulting in a reducing effect on heat transport. Similarly density and thermal conductivity of air tend to stabilize the flow and curtail the convective heat transport. Analysis shows that at higher Reynolds numbers the effect of viscosity is not dominant and thermal conductivity plays a significant role.

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ABBREVIATIONS

LES	Large eddy simulation
TLES	Thermal large eddy simulation
DNS	Direct numerical simulation
RANS	Reynolds averaged Navier-Stokes
IGW	Internal gravity waves
WALE	Wall adaptive local eddy viscosity
SGS	Subgrid scale
RMS	Root mean square
TKE	Turbulent kinetic energy
POD	Proper orthogonal decomposition
OB	Oberbeck Boussinesq
NOB	Non-Oberbeck Boussinesq
SSU	Stable stratification with uniform fluid properties
SSN	Stable stratification with non-uniform fluid properties
FCN	Forced convection with non-uniform fluid properties

NOTATION

a	Cross sectional area of jet
B_k	Production to turbulence by buoyancy
$B_{q,hot}$	Heat flux at hot wall
$B_{q,cold}$	Heat flux at cold wall
c	Speed of sound
C_d	Eddy viscosity model constant
C_p	Specific heat at constant pressure
C_v	Specific heat at constant volume
C_f	Skin friction coefficient
e	Internal energy per unit mass
E	Total energy per unit mass
F	Constant streamwise driving pressure gradient
F_B	Buoyancy force ratio between hot and cold walls
H	Total enthalpy per unit mass
k	Turbulent kinetic energy per unit mass
L_o	Ozmidov length scale
N	Buoyancy frequency
p	Kinetic pressure
p_0	Thermodynamic pressure
P_k	Production to turbulence by mean shear
q_j	Subgrid scale heat flux
q_j	Heat flux at wall
Q	Second invariant of the velocity gradient tensor
R	Specific gas constant
S_{ij}	Strain rate tensor

t	Time
T	Temperature
T_m	Mean temperature
u, v, w	Fluid velocities
u_τ	Shear velocity
U_b	Bulk velocity
U_m	Mean velocity
x, y, z	Streamwise, spanwise, and wall-normal direction
z^+	Distance from the wall in wall units $z^+ = zu_\tau/\nu$
Δ	Filter width
Δx^+	Grid spacing in wall units in streamwise direction
Δy^+	Grid spacing in wall units in spanwise direction
Δz^+	Grid spacing in wall units in normal direction
Δt	Time step
ΔT	Temperature difference between hot and cold wall
ΔT_{BL}	Temperature difference accross boundary layer
δ	Channel half width
δ_{ij}	Kronecker delta
μ	Dynamic viscosity of air
μ_{ref}	Reference dynamic viscosity
κ	Thermal conductivity of air
ν	Kinematic viscosity
ρ	Fluid density
ρ_{ref}	Reference fluid density
τ_{ij}	Shear stress
τ_{wall}	Wall shear stress
τ_η	Kolmogorov time scale
γ	Ratio of specific heat
β	Thermal expansivity

η	Kolmogorov length scale
ε	Turbulence dissipation rate
M	Mach number $M = U/C$
Re	Reynolds number $Re = \rho u \delta / \mu$
Re_τ	Shear Reynolds number $Re = \rho u_\tau \delta / \mu$
Pr	Prandtl number $Pr = \mu C_p / \kappa$
Pr_{ref}	Reference Prandtl number $Pr_{ref} = \mu_{ref} C_p / \kappa_{ref}$
Ri_τ	Shear Richardson number $Ri_\tau = Gr / Re^2$
Ri_g	Gradient Richardson number $Ri_g = N^2 / S^2$
Ri_f	Flux Richardson number $Ri_f = B_k / P_k$
R_θ	Temperature ratio between walls $R_\theta = T_{hot} / T_{cold}$
Gr	Grashof number $Gr = g \beta \Delta T (2\delta)^3 / \nu^2$
Nu	Nusselt number $Nu = \frac{\delta(\frac{\partial T}{\partial y})_w}{T_w - T_b}$
$()_b$	Bulk values
$()_m$	Mean values
$()_{rms}$	RMS values
$()_w$	Values at wall
$()_{ijk}$	Index number for spatial direction
$()_t$	Turbulent values
$()^+$	Non-dimensional values
$()^*$	Dimensional values
$()''$	Fluctuating Favre averaged values
$()'$	Fluctuating Reynolds averaged values