

LASER-DRIVEN PROTON ACCELERATION WITH STRUCTURED TARGETS AND IMPACT OF EXTERNAL MAGNETIC FIELD

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Laser-Driven Proton Acceleration With Structured Targets And Impact Of External Magnetic Field

by

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to the



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*Dedicated to the extraordinary love of my family, and the
constant belief of all those who did*

Certificate

This is to certify that the thesis entitled “**LASER-DRIVEN PROTON ACCELERATION WITH STRUCTURED TARGETS AND IMPACT OF EXTERNAL MAGNETIC FIELD**”, submitted by **IMRAN KHAN** to the Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** in PHYSICS, is a record of the original, bonafide research work carried out by him under our supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations related to the award of the degree.

The results contained in this thesis have not been submitted in part or in full to any other University or Institute for the award of any degree or diploma to the best of our knowledge.

Prof. Vikrant Saxena

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Date:

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Imran Khan

Abstract

This thesis investigates proton/ion acceleration through laser-plasma interactions, focusing on the impact of micron-sized grooved hydrocarbon targets with varying geometries, rectangular, triangular, and semi-circular, on the acceleration mechanism and the resulting energy spectra of accelerated protons. A comparative study reveals that a rectangular groove with a few-nanometer-thin rear wall and a width equal to the laser waist size yields the highest proton cut-off energy. Additionally, the influence of laser configuration was also examined, showing that although the oblique incidence of laser pulse enhances the protons' cut-off energy in the case of flat targets, this trend reverses for grooved targets, where maximum proton cut-off energy occurs with a single normal incidence pulse.

Furthermore, the impact of an externally applied kilo-Tesla magnetic field on electron and proton spectra has been studied for three laser pulse configurations: single normal pulse (SNP), single oblique pulse (SOP), and two oblique colliding pulses (TCP). Initial analyses with a flat target reveal a significant enhancement in proton cut-off energy across all configurations, with TCP proving to be the most effective setup. However, for a rectangular grooved (RG) target, the magnetic field has negative effect, resulting in a reduction in the protons' cut-off energy.

सारांश

यह शोध प्रबंध लेजर-प्लाज्मा अंतर्क्रियाओं के माध्यम से प्रोटॉन/आयन त्वरण का अध्ययन करता है, जिसमें आयताकार, त्रिकोणीय और अर्ध-वृत्ताकार ज्यामिति वाले माइक्रोन-आकार के खांचेदार हाइड्रोकार्बन लक्ष्यों के त्वरण प्रक्रिया और त्वरित प्रोटॉनों के ऊर्जा स्पेक्ट्रम पर प्रभाव पर ध्यान केंद्रित किया गया है। तुलनात्मक अध्ययन से पता चलता है कि कुछ नैनोमीटर पतली पिछली दीवार और लेजर व्यास आकार के बराबर चौड़ाई वाला आयताकार खांचा सर्वाधिक प्रोटॉन कट-ऑफ ऊर्जा उत्पन्न करता है। इसके अलावा, लेजर विन्यास के प्रभाव का भी विश्लेषण किया गया, जिससे यह स्पष्ट हुआ कि यद्यपि तिरछे कोण पर आपतित लेजर पल्स समतल लक्ष्यों के मामले में प्रोटॉन कट-ऑफ ऊर्जा को बढ़ाती है, परंतु खांचेदार लक्ष्यों के लिए यह प्रवृत्ति विपरीत हो जाती है, और अधिकतम प्रोटॉन कट-ऑफ ऊर्जा केवल एकल लंबवत आपतन पल्स के साथ प्राप्त होती है।

इसके अतिरिक्त, इलेक्ट्रॉन और प्रोटॉन स्पेक्ट्रम पर बाहरी रूप से लगाए गए किलो-टेस्ला चुंबकीय क्षेत्र के प्रभाव का अध्ययन तीन लेजर पल्स विन्यासों के लिए किया गया है: एकल सामान्य पल्स (SNP), एकल तिरछा पल्स (SOP), और दो तिरछी टकराने वाली पल्स (TCP)। समतल लक्ष्य के साथ प्रारंभिक विश्लेषण से ज्ञात होता है कि सभी विन्यासों में प्रोटॉन कट-ऑफ ऊर्जा में उल्लेखनीय वृद्धि होती है, जिसमें TCP संयोजन सबसे प्रभावी साबित होता है। हालांकि, एक आयताकार खांचेदार (RG) लक्ष्य के लिए, चुंबकीय क्षेत्र दुष्प्रभावी होता है, जिसके परिणामस्वरूप प्रोटॉन की कट-ऑफ ऊर्जा में कमी आती है।

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Abbreviations

MCF	M agnetic C onfinement F usion
ICF	I nertial C onfinement F usion
PFI	P roton F ast I gniation
BPM	B eam P ropagation M ethod
TNSA	T arget N ormal S heath A cceleration
RPA	R adiation P ressure A cceleration
HB	H ole B oring
LS	L ight S ail
RSIT	R elativistic S elf - I nduced T ransparency
CPA	C hirped P ulse A mplification
FA	F lat T arget
TG	T riangular G roove
SCG	S emi - C ircular G roove
RG	R ectangular G roove

Symbols

ϵ_0	dielectric constant/permittivity
k_B	Boltzmann constant
T_e	electron temperature
n_e	electron density
m_e	electron mass
ω_{pe}	plasma frequency
ω_{ce}	gyro frequency
ω_0	laser frequency
r_L	Larmor Radius
λ_d	wavelength
Λ_0	laser wavelength
λ_l	mean free path
P_{rad}	radiation pressure
I_0	laser intensity
a_0	normalized laser field
c	speed of light in vacuum
k_0	wavevector in free space