A THESIS ON
GENERAL THEORY OF DISCONTINUITIES IN NON EQUILIBRIUM GASDYNAMICS

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C E R T I F I C A T E

This is to certify that the thesis entitled 'General Theory of Discontinuities in Non Equilibrium Gasdynamics' which is being submitted by Mr. Rama Shanker for the award of Degree of Doctor of Philosophy (Mathematics) to the Indian Institute of Technology, Delhi, is a record of bonafide research work. He has worked for the last two and half years under my guidance and supervision.

The thesis has reached the standard fulfilling the requirements of the regulations relating to the degree. The results obtained in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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A C K N O W L E D G E M E N T S

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S Y N O P S I S

This thesis entitled 'General theory of discontinuities in Nonequilibrium Gas-Dynamics' deals with the problems of discontinuities in a non-relativistic, non-viscous and non-heat conducting relaxing fluids. In all it comprises of five chapters. Chapter I presents an introduction to the thesis. Chapter II deals with the differential effects of shock fronts. Chapter III discusses the geometry of the dissociative gas flows. Chapter IV is devoted to the study of the problem of growth and decay of sonic discontinuities in an ideal dissociating gas and chapter V is concerned with the study of the simple waves in one dimensional unsteady motion of an ideal dissociating gas. The contents of the various chapters are outlined below.

Chapter I - INTRODUCTION AND THE SCOPE OF THE PRESENT WORK :

In this chapter, a brief historical review of the previous studies related to the present work and the set of equations for a general class of relaxing fluids are given.

Chapter II - DIFFERENTIAL EFFECTS OF SHOCK FRONTS :

Under the assumption that the shock configuration to be a continuous differentiable function of coordinates, along which the velocity components, pressure, density and relaxation (mass-fraction for dissociation) behind the shock-surface are differentiable functions, we have in this chapter, studied the following topics:

(i) gradients of flow variables behind a shock

(ii) components of vorticity

(iii) curvature of an attached shock wave .

The complete chapter has been divided into two sections i.e. section-A and Section-B. In section-A we have studied the above problems in an ideal dissociative gas-dynamics, whereas in section-B, we have carried over the work of section-A to a general class of relaxing fluids.

The work of section-A is further divided into five parts. In part-I, following Thomas' Analysis, the curvature of an attached shock wave in plane steady flow of an ideal dissociating gas has been discussed. Parts II and III deal, using Truesdell's analysis, with the determination of the variation of flow quantities, curvature of stream lines and vorticity vector behind a curved shock waves in a three dimensional, steady as well as unsteady flow of an ideal dissociating gas. In parts IV and V, the problems of parts II and III have been re-examined by using Taub's analysis. It turns out that the effect of dissociation (or relaxation) is to decrease the density as well as pressure gradient, but increase the velocity gradient. Further, it is shown that, apart from the geometrical quantities, the components of vorticity depend through the shock strength on dissociation (or relaxation).

Chapter III - ON THE GEOMETRY OF DISSOCIATIVE GAS FLOWS:

This chapter is divided into three parts. In part-I, we have obtained the differential equations giving the direct determination of the stream line pattern for the plane steady flow of an ideal dissociating gas. While in part II , assuming the existence of these curves and surfaces we have obtained the variation of flow quantities along the stream lines, their principal normals and binormals in steady three dimensional ideal dissociative gas-dynamics, whereas in part III, we have carried over the study reported in part II, to a more general class of relaxing fluids. It turns out that the effect of dissociation is to decrease the vorticity components as well as, the pressure gradient along the stream line, whereas the trajectories of binormals lie on the surface of constant pressure.

Chapter IV - GROWTH AND DECAY OF SONIC DISCONTINUITIES IN AN IDEAL DISSOCIATING GAS:

In this chapter, differential equations for growth and decay of sonic discontinuities in an ideal dissociating gas have been formulated and for frozen flow, these equations have been integrated explicitly.

Chapter V - SIMPLE WAVES IN ONE DIMENSIONAL UNSTEADY DISSOCIATIVE GAS DYNAMICS:

This chapter is devoted to study the simple waves in one dimensional unsteady motion of an ideal dissociating gas and has the following important theorem:

'If the ratio of the difference of sensible internal energy and chemical bond energy to the dissociation energy per unit mass is such that it exceeds unity, then the particle speed decreases or increases as the dissociation variable increases, according as the flow is supersonic or subsonic'.

CONTENTS