DEVELOPMENT OF FINITE ELEMENT AND LATTICE BOLTZMANN SOLVERS FOR NONLINEAR DYNAMIC DEFORMATION WITH NON-UNIFORM STIFFNESS AND FLUID FLOW ANALYSIS

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by

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submitted

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I would like to dedicate this thesis to my family . . .

CERTIFICATE

This is to certify that the thesis entitled "**Development of finite element and lattice Boltzmann solvers for nonlinear dynamic deformation with non-uniform stiffness and fluid flow analysis**" being submitted by **Mr. Parvez Ahmad** to the **Indian Institute of Technology Delhi** for the award of the degree of **Doctor of Philosophy** is a bonafide record of original research work carried out by him under my supervision in conformity with rules and regulations of the institute. The results presented in this thesis have not been submitted, in part or in full, to any other University or Institute for the award of any degree or diploma.

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Parvez Ahmad

ABSTRACT

Fluid-structure interaction is a ubiquitous phenomenon with significant implications in both engineering and biological applications. In many instances, solvers for fluid-structure interactions employ a partitioned approach, where separate, standalone fluid and solid solvers are coupled together using a coupling algorithm. To accurately simulate real-life situations, the fluid and solid solvers must be general and free of restricting approximations. The most general solid solver should be capable of handling geometrically and materially nonlinear scenarios, allowing for large deformation and strain as well as large displacement and rotation, while also taking into account nonlinear constitutive relations.

The present work focuses on the development of a standalone fluid solver based on the Lattice Boltzmann Method and a standalone solid solver based on the Finite Element Method, both tailored for two-dimensional scenarios. These solvers were rigorously validated using results available in the literature and through comparison with commercial solvers like COMSOL Multiphysics. However, it is important to note that the coupling of the two solvers is beyond the scope of this work.

The developed fluid solver leverages the Lattice Boltzmann Method, an unconventional yet powerful computational fluid dynamics technique. This method employs a uniform Cartesian grid to simulate unsteady incompressible laminar fluid flow using the D2Q9 discrete velocity set. It utilizes Zou-He boundary conditions for the inlet and outlet, while also incorporating other straight and curved boundary conditions as required. The fluid solver's accuracy was validated by simulating flow around a cylinder with an attached downstream flap. The resulting data for overall drag and lift was found to be in excellent agreement with existing literature.

On the other hand, the developed structural solver is a time-dependent, geometrically-nonlinear large deformation solver based on the Finite Element Method. This solver employs the total Lagrangian formulation - an incremental approach used to solve the equilibrium equations. The dynamic response of the structure is captured using the standard Newmark time integration scheme, which has been adapted for the total Lagrangian formulation. The present structural solver has been validated for both static and dynamic cases. Different loading conditions, including volume, surface, and point loads, were simulated to verify the solver's wide applicability. Furthermore, the variation in the solver's accuracy was studied with respect to the number of elements, the aspect ratio of the elements, and the order of the elements. These studies also included an analysis of non-uniform stiffness, which was incorporated and validated within the solver.

In addition, the computational cost of the developed solver was evaluated with respect to the number and order of elements. The challenges associated with extending the solver to three-dimensional scenarios were also examined and discussed in detail which highlighted the increased complexity and computational demand that comes with three-dimensional simulations.

सार

फ्लूइड-स्ट्रक्चर इंटरैक्शन एक व्यापक घटना है जिसका महत्वपूर्ण प्रभाव इंजीनियरिंग और जैविक अनुप्रयोगों दोनों में होता है। कई मामलों में, फ्लूइड-स्ट्रक्चर इंटरैक्शन के लिए सॉल्वर एक विभाजित दृष्टिकोण अपनाते हैं, जहाँ अलग-अलग, स्वतंत्र फ्लूइड और सॉलिड सॉल्वर एक कपलिंग एल्गोरिदम का उपयोग करके जोड़े जाते हैं। वास्तविक जीवन की स्थितियों को सही ढंग से अनुकरण करने के लिए, फ्लूइड और सॉलिड सॉल्वर को सामान्य और प्रतिबंधात्मक अनुमानों से मुक्त होना चाहिए। सबसे सामान्य सॉलिड सॉल्वर को ज्योमेट्रिक और मटेरियल रूप से नॉनलिनियर परिदृश्यों को संभालने में सक्षम होना चाहिए, जिसमें बड़े विकृति और तनाव के साथ-साथ बड़े विस्थापन और घुमाव भी शामिल हों, और साथ ही नॉनलिनियर कॉन्स्टिट्यूटिव संबंधों को भी ध्यान में रखना चाहिए।

वर्तमान कार्य दो-आयामी परिदृश्यों के लिए तैयार लट्टिस बोल्ट्जमैन विधि पर आधारित एक स्वतंत्र फ्लूइड सॉल्वर और फिनाइट एलिमेंट विधि पर आधारित एक स्वतंत्र सॉलिड सॉल्वर के विकास पर केंद्रित है। इन सॉल्वरों को साहित्य में उपलब्ध परिणामों का उपयोग करके और COMSOL Multiphysics जैसे वाणिज्यिक सॉल्वरों के साथ तुलना करके सख्ती से मान्य किया गया था। हालाँकि, यह ध्यान देने योग्य है कि दोनों सॉल्वरों का युग्मन इस कार्य के दायरे से बाहर है।

विकसित फ्लूइड सॉल्वर लट्टिस बोल्ट्जमैन विधि का लाभ उठाता है, जो एक असामान्य लेकिन शक्तिशाली कम्प्यूटेशनल फ्लूइड डायनामिक्स तकनीक है। यह विधि D2Q9 डिस्क्रीट वेलोसिटी सेट का उपयोग करके अनस्टेडी, इनकॉम्प्रेसिबल लैमिनर फ्लूइड प्रवाह का अनुकरण करने के लिए एक समान कार्टेशियन ग्रिड को नियोजित करती है। यह इनलेट और आउटलेट के लिए ज़ो-ही सीमा शर्तों का उपयोग करती है, जबकि आवश्यकता अनुसार अन्य सीधे और घुमावदार सीमा शर्तों को भी शामिल करती है। फ्लूइड सॉल्वर की सटीकता को एक सिलेंडर के चारों ओर प्रवाह का अनुकरण करके मान्य किया गया

था जिसमें एक संलग्न डाउनस्ट्रीम फ्लैप था। कुल ड्रैग और लिफ्ट के लिए प्राप्त डेटा मौजूदा साहित्य के साथ उत्कृष्ट रूप से मेल खाता है।

दूसरी ओर, विकसित स्ट्रक्चरल सॉल्वर एक समय-निर्भर, ज्यामितीय-गैर-रेखीय बड़े विकृति सॉल्वर है जो फिनाइट एलिमेंट विधि पर आधारित है। यह सॉल्वर टोटल लाग्रांजियन फॉर्म्यूलेशन का उपयोग करता है, जो संतुलन समीकरणों को हल करने के लिए एक वृद्धिशील दृष्टिकोण है। स्ट्रक्चर की गतिशील प्रतिक्रिया को पकड़ने के लिए मानक न्यूमार्क समय एकीकरण योजना का उपयोग किया गया है, जिसे टोटल लाग्रांजियन फॉर्म्यूलेशन के लिए अनुकूलित किया गया है। सॉलिड सॉल्वर को स्थिर और गतिशील दोनों मामलों के लिए मान्य किया गया है। विभिन्न लोडिंग शर्तों, जिनमें वॉल्यूम, सतह और बिंदु भार शामिल हैं, को सॉल्वर की व्यापक प्रयोज्यता को सत्यापित करने के लिए अनुकरण किया गया था। इसके अतिरिक्त, तत्वों की संख्या, एलिमेंट्स के आयाम अनुपात और एलिमेंट्स के ऑर्डर के संबंध में सॉल्वर की सटीकता में भिन्नता का अध्ययन किया गया। इन अध्ययनों में गैर-एकरूप कठोरता का विश्लेषण भी शामिल था, जिसे सॉल्वर के भीतर शामिल और मान्य किया गया था।

इसके अलावा, सॉल्वर की गणनात्मक लागत को एलिमेंट्स की संख्या और ऑर्डर के संबंध में मूल्यांकन किया गया। सॉल्वर को तीन-आयामी परिदृश्यों तक विस्तारित करने से जुड़े चुनौतियों की भी जांच की गई और विस्तार से चर्चा की गई। इस विश्लेषण से तीन-आयामी अनुकरणों के साथ आने वाली बढ़ी हुई जटिलता और गणनात्मक मांग को उजागर किया गया, जो भविष्य के विकास और अनुकूलन के लिए संभावित क्षेत्रों में अंतर्दृष्टि प्रदान करता है।

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Nomenclature

Abbreviations

2D	Two-dimensional
3D	Three-dimensional
BGK	Bhatnagar Gross and Krook
CFD	Computational Fluid Dynamics
DAE	Differential Algebraic Equation
ELBM	Entropic Lattice Boltzmann Method
FDM	Finite Difference Method
FEM	Finite Element Method
FSI	Fluid-Structure Interaction
GL	Green-Lagrange
GPU	Graphics Processing Unit
LBE	Lattice-Boltzmann Equation
LBM	Lattice-Boltzmann Method
LSM	Lattice Spring Model
MKL	Math Kernel Library
MRT	Multiple Relaxation Time
PDE	Partial Differential Equation

SRT	Single Relaxation Time
TL	Total Lagrangian

Symbols

$\bar{\Delta}$	vector of velocity increments
$\bar{u}$	x-component of velocity increment
$\bar{v}$	y-component of velocity increment
Δt	time step
η	y-direction in natural co-ordinates
$\hat{f}_\alpha$	particle distribution function in moment space
ν	poisson ratio of the solid
ϕ_α	external force acting on particles in the α_{th} direction
Ψ	vector of shape function
ψ_i	i_{th} shape function
σ^s_{ij}	stress tensor for the solid
τ	relaxation time in LBM
$\tilde{f}_\alpha$	post-collision particle distribution function in LBM
ε^s_{ij}	strain tensor for the solid
ξ	x-direction in natural co-ordinates
ξ_i,η_i	location of gauss points
${}_0\eta_{ij}$	nonlinear part of the green-lagrange strain increment tensor

${}_0e_{ij}$	linear part of the Green-Lagrange strain increment tensor
B_L	linear strain-displacement transformation matrix
B_{NL}	nonlinear strain-displacement transformation matrix
C_i	i_{th} configuration of the deformed solid
c_s	velocity of sound
C_{ijkl}	material elasticity tensor
$d\,{}^0S$	differential surface element wrt reference configuration
$d\,{}^0V$	differential volume element wrt reference configuration
e_α	velocity vector in the α_{th} direction
E_{ij}	components of the green-lagrange strain tensor
E_{ij}	components of the second-piola kichoff stress tensor
f_α^{eq}	local equilibrium distribution function
f_i	body force per unit volume
f_α	particle distribution function in LBM
F_{ij}	components of the deformation gradient tensor
J	determinant of deformation gradient tensor
K_i	non-dimensional stiffness
n_{elem}	number of nodes per element
p_{elem}	order of element
t_i	traction vector per unit surface area

u_i	nodal values of the dependent variable
u_s	interpolating polynomial of p^{th}-order
w_α	weighing factor
w_i	weights of gauss points