

**ADAPTIVE FINITE ELEMENT ANALYSIS OF
AXISYMMETRIC AND PLANE STRAIN
SHEET METAL OPERATIONS**

by

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Department of Applied Mechanics

submitted

in fulfillment of the requirements of the Degree of
DOCTOR OF PHILOSOPHY

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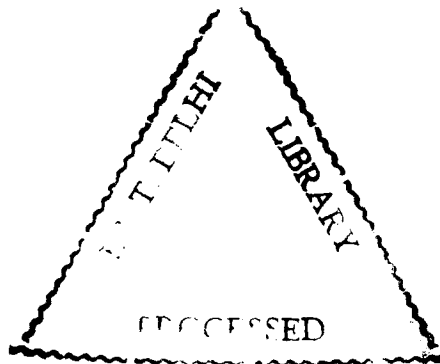
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CERTIFICATE

This is to certify that the thesis entitled '***Adaptive Finite Element Analysis of Axisymmetric and Plane Strain Sheet Metal Operations***' being submitted by *Mr. Mohd. Ahmed* to **the Indian Institute of Technology, Delhi** for the award of the Degree of Doctor of Philosophy is a record of bonafide research work carried out by him. *Mr. Mohd. Ahmed* has worked under my supervision and guidance and has fulfilled the requirements for the submission of this thesis, which to my knowledge has reached the requisite standard for the Doctor of Philosophy Degree.

The results contained in this thesis have not been submitted in part or full, to any other University or Institute for the award of any degree or diploma.



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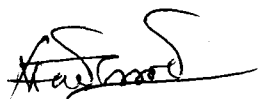
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ABSTRACT

The finite element method has emerged as an effective tool for simulating metal forming processes. However, the solution accuracy depends upon element choice, fineness of domain discretization, algorithm employed for solving system equations and the method employed for computing derivatives of state variables. Significant errors can creep in the computed solution unless special care is taken. The accuracy of numerical results can be enhanced through error estimation and adaptive mesh refinement. The present study deals with the performance of the recovery based adaptive finite element procedures in the context of finite element simulation of sheet metal operations. A systematic investigation of the effects of process parameters namely thickness of blank, and of certain parameters related to adaptive mesh refinement, namely accuracy limit and mesh refinement strategy has also been carried out. Shell and continuum elements have been used for discretizing the problem domain. Two "recovery" techniques namely node patch based recovery of derivatives and element patch based recovery for velocities have been employed. Both recovery techniques are based on the least square fitting of higher order polynomial to the computed values of stress and velocity.

A shell element based adaptive finite element model has been proposed for the analysis of axisymmetric sheet metal forming operations. The proposed shell formulation relies upon a velocity based flow approach and takes into consideration membrane, bending and shear effects. An adaptive finite element

code **AdSheet1** was developed by extensively modifying an existing two-dimensional code to suit the requirements of the proposed model. As the existing code was based on a continuum formulation using two-dimensional elements, many routines of the existing code had to be modified and some new routines had to be developed. For example, the mesh generation module, analysis module, and error estimation and adaptive refinement modules of the existing program were changed. The routines for incorporating strain rate matrix, jacobian matrix, modulus matrix and stiffness matrix were rewritten. Changes in the modules were necessitated for incorporating the sheet bending effect. Frictional effects have been included in the form of surface traction and stiffness contribution. The sheet thickness is updated at each time step to satisfy the incompressibility condition. Three noded line elements having three degrees of freedom at each node are used for discretization. The Zienkiewicz-Zhu (ZZ) error estimator has been employed to estimate error at element as well as global levels. The proposed procedure has been applied to finite element analysis of sheet metal operations and its performance has been studied. It is found that the proposed adaptive analysis enhances the efficiency and reliability of simulation. The proposed adaptive procedures are quite sensitive to the deformation pattern of the blank and can be used for prediction of plastic zones. Clearer and more accurate picture of deformation behaviour is obtained by using adaptive analysis in comparison to non-adaptive analysis. Effects of varying the blank thickness and refinement strategy have been studied. It is found that the effective strain in the deformed blank increases with increase of the blank thickness. However the

simulation time is more or less unaffected by changes in thickness of the metal sheet. It is also observed that accuracy of analysis is not significantly affected by the choice of the refinement strategy.

An existing two-dimensional finite element code FEM2D has been modified for the simulation of plane strain and axisymmetric sheet forming operations. It has been named as **AdSheet2**. As the available code was primarily designed for the analysis of bulk forming operations, it had to be modified to make it capable of simulating sheet metal forming operations. Also the capability of the code has been enhanced to cater to very fine meshes created through adaptive mesh refinement and improved visualisation of deformation pattern. New modules named as *load_ps* for modeling the blank holder and *eps_prjs* to compute components of stress and strain have been added. The modules related to prescribing the boundary conditions namely *tool_contact* and *tool_gen* have been enlarged to accept conditions imposed by the blank holder pressure. The program based upon the flow approach using rigid plastic and rigid visco-plastic models uses six noded triangular elements for mesh generation. A new mesh is generated whenever the previous mesh gets overly distorted. The von-Mises yield criterion has been employed. The incompressibility constraints have been introduced in the penalty form using reduced integration. Newton-Raphson method with linear line search technique is used for solving the non-linear equations. The Zienkiewicz-Zhu (ZZ) error estimator is used to estimate error at element as well as global levels. The proposed finite element code **AdSheet2** has been used for simulating stretching and drawing operations on sheet metal.

Performance of recovery based error estimation procedures has been analysed. It is found that efficiency and reliability of the predicted results is enhanced by adaptive procedures. It is also found that the recovery based adaptive procedures are sensitive to deformation behaviour. It is seen that the values of field variables are improved using adaptive refinement. The performance of velocity recovery based adaptive analysis appears to be better as compared to that of stress recovery based adaptive analysis. In other words, the element patch recovery of velocity improves the solution more than the nodal patch recovery of the derivatives. The effects of thickness of blank, choice of refinement strategy and error limit on the accuracy of analysis of sheet stretching operations are studied. It is found that the punch load increases with increase of blank thickness. There is no effect of choice of refinement strategy and error limit on the computed punch load. The influence of the process parameters, namely thickness of blank, ratio of radius of metal blank to radius of punch and friction at the blank-punch and blank-die interface, in deep drawing operations has been studied. The numerical results show that the punch load is increased with the increase of blank thickness, it is decreased with the increase of radius ratio and is not materially affected by frictional condition at interfaces.

In the author's mind, the chief contribution of the present work lies in the development of finite element codes for adaptive analysis of sheet metal operations and the evaluation of performance of recovery based adaptive finite element procedures in the context of finite element analysis of sheet metal operations.

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