

NONLINEAR STATIC AND DYNAMIC ANALYSES OF THIN-WALLED COMPOSITE STRUCTURES

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**DEPARTMENT OF APPLIED MECHANICS
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NONLINEAR STATIC AND DYNAMIC ANALYSES OF THIN-WALLED COMPOSITE STRUCTURES

by

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Submitted

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CERTIFICATE

This is to certify that the thesis entitled, '**Nonlinear Static and Dynamic Analyses of Thin-Walled Composite Structures**', which is being submitted by **Ms. Emarti Kumari** to the Indian Institute of Technology Delhi in fulfilment of the requirements for the award of degree of **Doctor of Philosophy** embodies original research work done by her under my supervision. In my opinion, the thesis work meets the requisite standards and the candidate is worthy of consideration for the degree of Doctor of Philosophy in accordance with the regulations of the institute. The contents of this thesis have not been submitted to any other University or Institute for the award of the degree or diploma.

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Abstract

The present thesis deals with the nonlinear static and dynamic behaviors of thin-walled structures made of isotropic and laminated composite materials. *Sixteen-node degenerated shell element*, based on the total Lagrangian formulation for *large displacement* (Green-Lagrange strain tensor) and *large rotation of the normal vector* is employed here to model few common structural elements, such as, isolated plates, web plates of built-up sections (assembled plate structures), cantilever twisted panels and curved shell panels. Attempt is made to investigate some characteristic features of the nonlinear behaviors of such *flexible* and *shear-deformable* thin-walled structures.

The nonlinear stability behavior of built-up beams (*e.g.*, box-beam, I-beam and C-section) is investigated at the beginning. The *state-of-stresses* in the individual plates (*e.g.*, web panel) of such assembled plate structures are significantly different than isolated flat panels. Here, the stability behavior of the web panels with or without restraints from the *idealistic stress-free flanges* are studied under four different theoretical *states-of-stresses*, *i.e.*, (a) pure shear with parabolic distribution through the height of the web panel, (b) in-plane bending, (c) shear and in-plane bending and (d) combined stresses involving in-plane shear, bending and compression. Then, the buckling behavior of the idealistic compression flange is also investigated considering restraints from the *idealistic stress-free web plates*. The correlation between the buckling loads of such idealistic web and flange panels and the complete built-up sections provide enhanced understanding on the stability characteristics of assembled plate structures.

Next, the nonlinear free vibration behavior of cantilever plates are considered, where, the accuracy of the solution depends on several factors, such as, the *structural model*, *discretization technique* (assumed space-mode analytical solution or numerical discretization)

and finally the *solution procedure*. The present higher-order element, based on Green-Lagrange nonlinearity for large displacement along with large rotation of the normal vector is employed to study the large-amplitude *individual-mode* free flexural vibration behavior of the first three *bending-modes* and the first *torsion-mode* of flat and pre-twisted cantilever panels. The effect of transverse shear deformation on the nonlinear behavior of moderately thick, isotropic and laminated composite cantilever plates is explored. Next, the effect of rotation speed on the nonlinear free flexural and torsional vibration behavior of flat and pre-twisted cantilever panels is also taken up for investigation.

Finally, the nonlinear transient response of *flat and curved panels* under extreme loading condition, *e.g.*, thermal shock due to sudden heating and mechanical shock during blast loading is investigated. For this purpose, the relative performances of several *step-by-step implicit time integration techniques* are examined. Thereafter, the effect of several geometrical (*radius-to-span* and *span-to-thickness* ratio) and material parameters (isotropic and laminated composites with different ply-orientations) on the response of moderately thick curved panels under thermal and mechanical shock are studied in detail.

सार

वर्तमान थीसिस आइसोट्रोपिक और लैमिनेटेड मिश्रित सामग्रियों से बने पतली दीवारों वाली संरचनाओं के गैर-स्थिर स्थिर और गतिशील व्यवहार से संबंधित है। सोलह-नोड पतित खोल तत्व, बड़े विस्थापन (ग्रीन-लैग्रेग स्ट्रेन टेन्सर) के लिए कुल लैग्रैन्ज्यूएशन फॉर्मूले पर आधारित है, और सामान्य वेक्टर के बड़े रोटेशन को कुछ सामान्य संरचनात्मक तत्वों, जैसे कि अलग-थलग प्लेट्स, वेब प्लेट्स के निर्माण के लिए नियोजित किया गया है। अप सेक्शन (इकट्टे प्लेट स्ट्रक्चर), कैंटिलीवर ट्विस्टेड पैनल और कवर्ड शेल पैनल। ऐसी लचीली और कतरनी-विकृतिपूर्ण पतली दीवारों वाली संरचनाओं के गैर-व्यवहार वाले व्यवहार की कुछ विशेषताओं की जांच करने का प्रयास किया जाता है।

बिल्ट-अप बीम (जैसे, बॉक्स-बीम, आई-बीम और सी-सेक्शन) के नॉनलाइनियर स्थिरता व्यवहार की शुरुआत में जांच की जाती है। इस तरह के इकट्टे प्लेट संरचनाओं की व्यक्तिगत प्लेटों (जैसे, वेब पैनल) में राज्य के तनाव अलग-थलग पड़े हुए पैनल की तुलना में काफी भिन्न होते हैं। यहां, आदर्शवादी तनाव-मुक्त फ्लैज के या बिना प्रतिबंधों के वेब पैनलों की स्थिरता व्यवहार का अध्ययन चार अलग-अलग सैद्धांतिक अवस्थाओं के अंतर्गत किया जाता है, अर्थात्, (a) पुरे शियर विथ पैराबोलिक डिस्ट्रीब्यूशन थ्रू थे हाइट ऑफ़ थे वेब पैनल। (बी) इन-प्लेन बेन्डिंग, (सी) शियर एंड इन-प्लेन बेन्डिंग एंड (डी) कंबाइंड स्ट्रेसेस इन्वोल्विंग इन-प्लेन शियर, बेन्डिंग एंड कम्प्रेसन। थे, थे बुखलिंग बिहेवियर ऑफ़ थे इडलिस्टिक कम्प्रेसन फ्लैज इस आल्सो इन्वेस्टीगेटेड कन्सिडरिंग रेस्ट्रॉन्ट्स फ्रॉम थे इडलिस्टिक स्ट्रेस-फ्री वेब प्लेट्स। इस तरह के आदर्शवादी वेब और निकला हुआ किनारा के बढ़ते भार और पूर्ण निर्मित वर्गों के बीच सहसंबंध इकट्टे प्लेट संरचनाओं की स्थिरता विशेषताओं पर बढ़ी हुई समझ प्रदान करता है।

इसके बाद, कैंटिलीवर प्लेटों के नॉनलाइनर मुक्त कंपन व्यवहार को माना जाता है, जहां, समाधान की सटीकता कई कारकों पर निर्भर करती है, जैसे कि संरचनात्मक मॉडल, विवेकाधिकार तकनीक (अंतरिक्ष-मोड विश्लेषणात्मक समाधान या संख्यात्मक विवेकाधिकार) और अंत में समाधान प्रक्रिया।। वर्तमान उच्च-क्रम तत्व, सामान्य वेक्टर के बड़े रोटेशन के साथ-साथ बड़े विस्थापन के लिए ग्रीन-लैग्रेग नॉनलाइनरिटी पर आधारित है, जो पहले तीन बेन्डिंग-मोड और पहले टॉरशन के बड़े-आयाम वाले इंडिविजुअल-मोड फ्लेक्सुरल कंपन व्यवहार का अध्ययन करने के लिए कार्यरत है। थे इफ़ेक्ट ऑफ़ ट्रांसवर्स शियर डेफोर्मेशन पर थे नॉनलीनर बिहेवियर ऑफ़ मोडेराटेली थिक, इसोट्रोपिक एंड लैमिनेटेड कम्पोजिट कैंटिलीवर प्लेट्स इस एक्सप्लोरेड। इसके बाद, फ्लैट और प्री-ट्विस्टेड कैंटिलीवर

पैनल के नॉनलाइन फ्री फ्लेक्सुरल और टॉर्सनल कंपन व्यवहार पर रोटेशन की गति का प्रभाव भी जांच के लिए उठाया गया है।

अंत में, अत्यधिक लोडिंग स्थिति के तहत फ्लैट और घुमावदार पैनलों की नॉनलाइनियर क्षणिक प्रतिक्रिया, जैसे, अचानक हीटिंग के कारण थर्मल झटका और ब्लास्ट लोडिंग के दौरान यांत्रिक झटके की जांच की जाती है। इस उद्देश्य के लिए, कई चरण-दर-चरण अंतर्निहित समय एकीकरण तकनीकों के सापेक्ष प्रदर्शन की जांच की जाती है। इसके बाद, थर्मल और मैकेनिकल झटके के तहत मामूली मोटी वक्र पैनलों की प्रतिक्रिया पर कई ज्यामितीय (त्रिज्या-से-अवधि और अवधि-से-मोटाई अनुपात) और सामग्री मापदंडों (अलग-अलग प्लार्ड-ओरिएंटेशन वाले आइसोट्रोपिक और लैमिनेटेड कंपोजिट) के प्रभाव का अध्ययन किया जाता है, विस्तार से।

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