ON LOTS AND GLOTS

BY

PRADIP KUMAR SACHAR

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CERTIFICATE

This is to certify that the thesis entitled 'ON LOTS AND GLOTS' being submitted by Mr. Pradip Kumar Sachar to the Indian Institute of Technology, Delhi, for the award of the degree of Doctor of Philosophy in Mathematics, is a record of bonafide research work carried out by him under my guidance and supervision for the last four and half years. To the best of my knowledge it has reached the standard, fulfilling the requirements of the regulations relating to the degree.

It is further certified that the results contained in this thesis have not been submitted to any other university or Institute for the award of any degree or diploma.

Wagish Shukla

(Wagish Shukla)
Assistant Professor
Department of Mathematics
Indian Institute of Technology
Hauz Khas, New Delhi-110016.

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Pradip Kumar Sachar
(Pradip Kumar Sachar)

CONTENTS

ABSTRACT

Linearly ordered topological spaces have been studied for more than fifty years now. The order-structure ensures a topological structure rich enough to support fruitful investigation and the morphisms bear enough load to observably distinguish themselves from merely order-preserving or merely continuous maps. LOTS, the category of linearly ordered topological spaces and GLOTS, the category of generalized linearly ordered topological spaces are the two main categories studied in the present investigation, though our approach is only occasionally functorial.

The contents of the thesis which consists of five chapters, can be described as follows:

The first chapter centers around the Dedekind completion of a GLOTS X. We characterize Dedekind complete LOTS abbreviated as DC LOTS, provide the Dedekind completion of GLOTS, prove epireflectively of DC LOTS, and investigate it, in general, in a fashion parallel to that of Stone-Cech compactification. We also discuss ordered extremal disconnectedness. Another compactness, (E,BE)-compactness has been introduced and studied in this chapter.

In chapter two, we have noted some properties of lexicographic products and obtained the conditions under which such a product of DC LOTS's again turns out to be a DC LOTS. We also discuss projection maps. The lexicographic product of

GLOTS's has been defined and studied, inspired by a construction of Purisch which we have modified to suit our purpose. This chapter also includes a section on 'Ordered Sums'.

Chapter three is an assortment consisting of some results of a categorical nature and some links with lateral investigations into the order-topology relationship e.g. those carried out by Nachbin. We also discuss functoriality and otherwise of some correspondences and the nature of the ordered graph of a function. We also look at the real line as a LOTS and in this process locate some of its standard properties in a DC LOTS, a LOTS or a GLOTS.

In chapter four, we collect some relevant information obtained by looking at the Souslin conjecture. We have proved a standard result whose proof we were unable to locate in the literature, observed and verified the conjecture in a linearly ordered topological group, and derived certain topological consequences of the hypothesis of the Souslin conjecture. We close the chapter by a result on the order homogeneity of a connected metrizable LOTS.

Chapter five, the last chapter of the present thesis, is in the nature of an appendix surveying some of the recent work on LOTS and GLOTS.

The original plan of the thesis included a section on examples and counter examples. In the final version however, that section has been dropped as it was discovered that the observations were available in the literature in equivalent versions.