

FINITE ELEMENT ANALYSIS OF MOVING LOAD ON BALLASTED AND BALLASTLESS RAIL TRACKS

Pranjal Mandhaniya



**DEPARTMENT OF CIVIL ENGINEERING
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by

Pranjal Mandhaniya

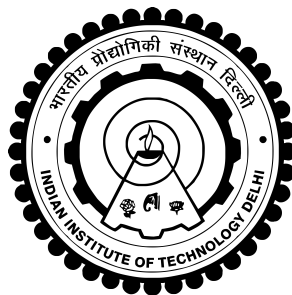
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CERTIFICATE

This is to certify that the thesis titled **Finite element analysis of moving load on ballasted and ballastless rail tracks**, submitted by **Mr. Pranjal Mandhaniya**, to the Indian Institute of Technology, Delhi, for the award of the degree of **Doctor of Philosophy**, is a bonafide record of the research work done by him under our supervision and guidance. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Dr. J.T. Shahu
Professor
Dept. of Civil Engineering
IIT Delhi, 110016

Dr. Sarvesh Chandra
Former Professor
Dept. of Civil Engineering
IIT Kanpur, 208016

New Delhi
March, 2023

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(Pranjal Mandhaniya)

ABSTRACT

High-speed rail transportation requires careful design of a rail track. The design of a railway track is governed by the load-bearing capacity and maximum possible speed of transportation. The capacity of a high-speed rail tracks is generally limited by its critical speed. It is the speed at which the vibration occurs at the largest magnitude. Thus, its identification is necessary to prevent derailments and damage to the rail tracks.

The present study attempted a three-dimensional finite element analysis of a typical ballasted rail track under a single moving wheel load at different speeds to evaluate the critical speed and to assess the dynamic impact factors at subcritical speeds. Dynamic impact factor (DIF) is the ratio of dynamic and static wheel load. It determines the effect of dynamic load (mostly under subcritical speeds) on a rail track based on several empirical formulations given in the literature. These formulations are developed under a low-speed range of about 50 km/h on old tracks. Thus, their reliability for rail tracks under higher speeds up to 200 km/h needs to be validated. The study continues to a moving load analysis on a ballasted rail track resting over different geometric combinations of the geosynthetic reinforced earth (GRE) embankment and retaining wall to analyse the effect of reinforcing agents on embankment section configurations, including their approximate construction cost. The present study also covers preliminary work on moving load finite element analysis on embedded track system (ETS) model, which is a ballastless rail track system.

Ballast, subballast, and subgrade layers of the track were modelled as elastoplastic ma-

terial with material damping and radiation damping (infinite layers). Track response were calculated in the form of stress, displacement, velocity, and acceleration responses. Different parameters were analysed for suitability towards finding the critical behaviour, such as the method of applying moving load, material model of primary load-carrying layer (ballast), effect of boundaries, and the track responses extracted from the output databases. Parametric studies were performed on material damping and stiffness of track substructure layers to see their effect on the track's dynamic response. The effect of shear strength parameters (cohesion and friction) of the subgrade was also analysed to examine their effect on the critical behaviour of the rail track.

For the assessment of dynamic impact factors, the motion of a pressure-loaded areal patch was simulated using the VDLOAD subroutine in ABAQUS. Parametric studies on train speed, track modulus, and wheel diameter were performed. Regression analysis was performed on the finite element results, and a DIF formulation was developed as a function of speed and track modulus. The study shows that a combination of vertical velocity, stress, and acceleration trends can be used to identify the critical speed of the rail track, which was found as 350 km/h for the present study. It was found that the Youngs moduli of substructure did not show direct proportionality with the critical speed. The damping ratio has a small effect over the critical speed, while the increase in shear strength of the subgrade increases the critical speed.

Regarding the DIF assessments, it was inferred that the difference between the existing DIF formulae and the present study increases with speed. The relationship calculated from the numerical study suggested a higher DIF than the existing relationships in the literature. At higher speeds (>150 km/h), the elastoplastic behaviour of the rail track causes higher vibration as compared to lower speeds (<50 km/h) where dynamic vibrations are generally

under elastic range.

The analysis of GRE structures under moving load showed that the ground acceleration decreases by 50 % with geosynthetics and facing walls in the speed range of 100 to 400 km/h. It was also inferred that the bulk mass of earthen embankments is not enough to counter large vibrations. The study on ballastless ETS model suggested that the stiffness and thickness of concrete bearing layer will affect the rail track response the most.

संक्षेप

हाई-स्पीड रेल परिवहन के लिए रेल ट्रैक के सावधानीपूर्वक डिजाइन की आवश्यकता होती है। रेलवे का डिजाइन ट्रैक भार वहन क्षमता और परिवहन की अधिकतम संभव गति से नियंत्रित होता है। हाई-स्पीड रेल पटरियों की क्षमता आम तौर पर इसकी महत्वपूर्ण गति से सीमित होती है। यह है जिस गति से कंपन सबसे बड़े परिमाण में होता है। ऐसे में इसकी पहचान है पटरी से उतरना और रेल पटरियों को नुकसान को रोकने के लिए आवश्यक है।

वर्तमान अध्ययन ने एक विशिष्ट के त्रि-आयामी परिमित तत्व विश्लेषण का प्रयास किया मूल्यांकन करने के लिए अलग-अलग गति से एक ही गतिमान पहिया भार के तहत गिट्टी रेल ट्रैक महत्वपूर्ण गति और उप-महत्वपूर्ण गति पर गतिशील प्रभाव कारकों का आकलन करने के लिए। गतिशील इम्पैक्ट फैक्टर (DIF) डायनेमिक और स्टैटिक व्हील लोड का अनुपात है। यह प्रभाव निर्धारित करता है | कई अनुभवजन्य के आधार पर एक रेल ट्रैक पर गतिशील भार (ज्यादातर सबक्रिटिकल गति के तहत)। साहित्य में दिए गए फॉर्मूलेशन। ये फॉर्मूलेशन कम गति के तहत विकसित किए गए हैं पुराने ट्रैक पर लगभग 50 किमी/घंटा की रेंज। इस प्रकार, उच्च के तहत रेल पटरियों के लिए उनकी विश्वसनीयता 200 किमी/घंटा तक की गति को सत्यापित करने की आवश्यकता है। मूविंग लोड विश्लेषण के लिए अध्ययन जारी है जियोसिंथेटिक के विभिन्न ज्यामितीय संयोजनों पर आराम करने वाली गिट्टी वाली रेल पटरी पर सुदृढीकरण के प्रभाव का विश्लेषण करने के लिए प्रबलित पृथ्वी (जीआरई) तटबंध और दीवार को बनाए रखना तटबंध खंड विन्यास पर एजेंट, उनकी अनुमानित निर्माण लागत सहित। वर्तमान अध्ययन में मूविंग लोड परिमित तत्व विश्लेषण पर प्रारंभिक कार्य भी शामिल है एम्बेडेड ट्रैक सिस्टम (ETS) मॉडल, जो एक गिट्टी रहित रेल ट्रैक सिस्टम है।

गिट्टी, सबबॉलास्ट, और ट्रैक की सबग्रेड परतों को इलास्टोप्लास्टिक सामग्री के रूप में सामग्री भिगोना और विकिरण भिगोना (अनंत परतें) के रूप में तैयार किया गया था। ट्रैक प्रतिक्रिया की गणना तनाव, विस्थापन, वेग और त्वरण प्रतिक्रियाओं के रूप में की गई थी। महत्वपूर्ण व्यवहार को खोजने के लिए उपयुक्तता के लिए विभिन्न मापदंडों का विश्लेषण किया गया था, जैसे मूविंग लोड को लागू करने की विधि, प्राथमिक लोड-ले जाने वाली परत (गिट्टी) का सामग्री मॉडल, सीमाओं का प्रभाव और आउटपुट डेटाबेस से निकाले गए ट्रैक

प्रतिक्रियाएं। ट्रैक की गतिशील प्रतिक्रिया पर उनके प्रभाव को देखने के लिए ट्रैक सबस्ट्रक्चर परतों की सामग्री नमी और कठोरता पर पैरामीट्रिक अध्ययन किए गए थे। रेल ट्रैक के महत्वपूर्ण व्यवहार पर उनके प्रभाव की जांच करने के लिए सबग्रेड के अपरूपण शक्ति मापदंडों (सामंजस्य और घर्षण) के प्रभाव का भी विश्लेषण किया गया था।

गतिशील प्रभाव कारकों के आकलन के लिए, ABAQUS में VDLOAD सबरूटीन का उपयोग करके दबाव-भारित क्षेत्र पैच की गति का अनुकरण किया गया। ट्रेन की गति, ट्रैक मापांक और पहिया व्यास पर पैरामीट्रिक अध्ययन किया गया। परिमित तत्व परिणामों पर प्रतिगमन विश्लेषण किया गया था, और गति और ट्रैक मापांक के कार्य के रूप में एक डीआईएफ सूत्रीकरण विकसित किया गया था। अध्ययन से पता चलता है कि रेल ट्रैक की महत्वपूर्ण गति की पहचान करने के लिए ऊर्ध्वाधर वेग, तनाव और त्वरण प्रवृत्तियों के संयोजन का उपयोग किया जा सकता है, जो वर्तमान अध्ययन के लिए 350 किमी/घंटा के रूप में पाया गया था। यह पाया गया कि यंग के अवसंरचना ने महत्वपूर्ण गति के साथ प्रत्यक्ष आनुपातिकता नहीं दिखाई। भिगोना अनुपात का महत्वपूर्ण गति पर एक छोटा प्रभाव पड़ता है, जबकि सबग्रेड की कतरनी ताकत में वृद्धि होती है क्रांतिक गति को बढ़ाता है।

डीआईएफ आकलन के संबंध में, यह अनुमान लगाया गया था कि मौजूदा के बीच का अंतर डीआईएफ सूत्र और वर्तमान अध्ययन गति के साथ बढ़ता है। से संबंध की गणना संख्यात्मक अध्ययन ने साहित्य में मौजूदा संबंधों की तुलना में उच्च डीआईएफ का सुझाव दिया। उच्च गति (>150 किमी/घंटा) पर, रेल ट्रैक का इलास्टोप्लास्टिक व्यवहार उच्च का कारण बनता है कम गति की तुलना में कंपन (<50 किमी/घंटा) जहां गतिशील कंपन आम तौर पर होते हैं। मूविंग लोड के तहत जीआई संरचनाओं के विश्लेषण से पता चला है कि जमीनी त्वरण 100 से 400 की गति सीमा में जियोसिंथेटिक्स और सामना करने वाली दीवारों के साथ 50% की कमी किमी/घं. यह भी अनुमान लगाया गया था कि मिट्टी के तटबंधों का विशाल द्रव्यमान इसके लिए पर्याप्त नहीं है बड़े कंपन का मुकाबला करें। गिट्टी रहित ईटीएस मॉडल पर अध्ययन ने सुझाव दिया कि कठोरता और कंक्रीट असर परत की मोटाई रेल ट्रैक की प्रतिक्रिया को सबसे अधिक प्रभावित करेगी।

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List of symbols

c_d	Dilatation wave speed
C	Material damping matrix
D	Diameter of wheel
F	Force matrix
K	Stiffness matrix
L_d	Smallest mesh element size
M	Mass matrix
$u(t)$	Time-dependent displacement matrix
$u'(t)$	First derivative of time-dependent displacement matrix
$u''(t)$	Second derivative of time-dependent displacement matrix
U'	Vertical displacement or displacement magnitude
U	Track modulus
V'	Vertical velocity or velocity magnitude
V	Translation speed of wheel
V_R	Rayleigh wave velocity
V_s	Shear wave velocity
α	Mass proportional damping coefficients
β	Stiffness proportional damping coefficients
Δt_{min}	Minimum time increment size in an explicit integration
ξ	Damping constant of the material
ω_1	First resonant frequency of the structure
ω_2	Applied highest load frequency of the structure