COUPLED EXACT 3D SOLUTIONS AND 2D ZIGZAG THEORY FOR THERMO-ELECTRO-MECHANICAL ANALYSIS OF HYBRID PIEZOELECTRIC PLATES

by

G. G. SRINIVAS ACHARY

Department of Applied Mechanics

Submitted

in fulfilment of the requirements of

the degree of

Doctor of Philosophy

to the

Indian Institute of Technology, Delhi

July, 2005

Hybrid piezoelectric f ...
Plates

To

My Parents

Certificate

This is to certify that the thesis entitled ***"Coupled Exact 3D Solutions and 2D Zigzag Theory for Thermo-Electro-Mechanical Analysis of Hybrid Piezoelectric Plates"*** *being submitted by Mr. G. G. Srinivas Achary to the Indian Institute of Technology, Delhi for the award of the degree of Doctor of Philosophy in Applied Mechanics is a record of original bonafide research work carried out by him under my supervision and guidance. The thesis work, in my opinion, has reached the requisite standard fulfilling the requirements for the degree of Doctor of Philosophy.*

The results contained in this thesis have not been submitted in part or full to any other University or Institute for the award of any degree or diploma.

(Dr. S. Kapuria)
Associate Professor
Deptt. of Applied Mechanics
Indian Institute of Technology
New Delhi - 110016

Acknowledgements

I express my heartiest gratitude to Dr. S. Kapuria who, by his able guidance and meticulous supervision, set the sense of direction to this work and provided active support and kind help at every stage of the work. I am deeply indebted to Mr. V. Y. Salpekar who gave me the permission, support and encouragement to carry out this research work. I am also grateful to Dr. S. Sengupta and Prof. P. C. Dumir for their motivation, encouragement and support during the work.

My organisation, Engineers India Limited, has allowed me to do this work and the Engineering and Technology Development (ETD) Department has provided me with necessary support and environment, for which I am grateful to all concerned. Specifically, I am thankful to Mr. K. P. Sahu of ETD Department for his encouragement and support. I would like to extend very special thanks to my colleagues, Mr. K. Gurumurthy and Mr. Sudip Paul for their help, support and assistance during this work.

I wish to give my thanks to Mr. Naushad Alam for his help during the course of this work. I also want to thank all others who have directly or indirectly helped me during the course of this work.

Finally, many hours spent in carrying out this work were hours that would otherwise have been spent with my parents and wife Krishna. I do not have words to express my gratitude for their love, patience, inspiration and sacrifices.

G. G. Srinivas Achary

Abstract

A new efficient coupled geometrically nonlinear zigzag theory is presented for piezoelectric hybrid composite and sandwich plates under electrothermomechanical loading. The thermal field is approximated as piecewise linear across a suitable number of subdivisions in the layers to enable accurate modelling of the actual thermal profile across the thickness. The electric potential is similarly approximated as piecewise linear across a number of subdivisions in the piezoelectric layers. Both the inplane and the transverse electric fields are considered. The transverse displacement is approximated as a combination of a term independent of the thickness coordinate with additional sublayerwise quadratic terms that explicitly account for the transverse normal strain due to the transverse electric field and the thermal field. The inplane displacements are approximated as a combination of a global third order variation across the thickness with an additional layerwise linear variation. The displacement field is finally expressed in terms of only five primary displacement variables, electric potential variables and the thermal field by enforcing exactly the conditions of zero transverse shear stresses at the top and bottom surfaces and the conditions of their continuity at the layer interfaces. The number of the primary displacement variables is the same as in the smeared third order theory (TOT). The geometric nonlinearity is included in the sense of Von Karman. The governing coupled equations of motion, charge balance equations and boundary conditions are derived from the extended Hamilton's principle. The theory can model open and closed circuit boundary conditions. This zigzag theory, which accounts for the transverse extensibility due to temperature rise in the approximation of deflection, is a new theory even for the elastic laminated plates for thermal load.

A new smeared coupled geometrically nonlinear consistent third order shear deformation theory (CTOT) is developed for hybrid plates with a cubic approximation for the inplane displacements, uniform deflection across the thickness and piecewise linear approximation of the electric potential and the temperature-rise. Unlike the existing TOT, the conditions of zero transverse shear stresses at the bottom and the top are exactly satisfied including the contribution of the electric potential.

To assess the accuracy of the new theories, exact three-dimensional (3D) coupled piezoelasticity benchmark solutions are developed for i) steady state response of simply-supported rectangular

cross-ply hybrid plates under harmonic load with and without damping, and ii) buckling and free vibration of simply-supported rectangular symmetric cross-ply hybrid plates under initial inplane electrothermomechanical loads. The latter solution includes the effect of initial transverse normal strain, which is significant for thermoelectric loading, and accounts for the electric boundary conditions.

Analytical Fourier series solutions are obtained for the new 2D theories for simply-supported hybrid cross-ply rectangular plates for the linear static response under electrothermomechanical load, for the natural frequencies and for the steady state forced response under harmonic electromechanical load. The nonlinear theory is used to obtain the buckling and free vibration response of initially stressed simply-supported symmetric cross-ply hybrid rectangular plates under inplane electrothermomechanical load, accounting for open and closed circuit boundary conditions. A comprehensive assessment of the zigzag theory is made for static electrothermomechanical response, dynamic electromechanical response, and buckling and free vibration response under initial electrothermomechanical load, by comparing its results for simply-supported cross-ply rectangular plates with the exact 3D piezothermoelasticity solutions. The zigzag theory results are also compared with the TOT and the CTOT, which have the same efficiency as the zigzag theory. For this purpose, hybrid test plates with highly inhomogeneous lay-up; plates with symmetric and antisymmetric cross-ply composite substrates; and plates with sandwich substrate are analysed for various electrothermomechanical loads. The zigzag theory is found to yield generally very accurate results for the global response parameters as well as for the through-the-thickness variations of displacements, electric potential and stresses. In contrast, the CTOT and the TOT have been shown to yield highly inaccurate results for moderately thick and even thinner plates with inhomogeneous composite and sandwich substrates.

The major contributions of this work are i) development of the exact 3D piezothermoelasticity solution for buckling and free vibration analysis of initially stressed plates including the effect of initial transverse normal strain, ii) development of the efficient zigzag theory for static, dynamic and buckling electrothermomechanical analysis of hybrid plates, which considers electric field along inplane and transverse directions, and accounts for the transverse extensibility due to the piezoelectric and thermal strain in the thickness direction, and iii) a comprehensive assessment of this new theory in direct comparison with the exact 3D piezothermoelasticity solutions for a large variety of cases. This theory is as efficient as the TOT and yet has been shown to be generally very accurate for moderately thick hybrid plates and far superior to the latter.

Contents