

# **STUDIES ON UNDERWATER DIRECTIONAL ACOUSTIC SENSOR**

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**CENTRE FOR APPLIED RESEARCH IN ELECTRONICS  
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# STUDIES ON UNDERWATER DIRECTIONAL ACOUSTIC SENSOR

by

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**CENTRE FOR APPLIED RESEARCH IN ELECTRONICS**

Submitted

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to the



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*Dedicated to*

***Mother India***

# Certificate

This is to certify that the thesis entitled “**Studies on Underwater Directional Acoustic Sensor**” being submitted by **Mr. Bipin Kumar** to the Centre for Applied Research in Electronics, Indian Institute of Technology Delhi, for the award of the degree of **Doctor of Philosophy** is the record of bonafide research work carried out by him under my supervision. In my opinion, the thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted either in part or in full to any other university or institute for the award of any degree or diploma.



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Bipin Kumar

# Abstract

Acoustic sensors that preferentially detect sound from a given direction over other directions are known as directional acoustic sensors. Several underwater directional acoustic sensors have been investigated in the thesis. Analytical, simulation and experimental studies have been conducted, and prototypes of first-order and second-order directional acoustic sensors have been developed. Additionally, an analytical expression has been derived to determine the weights of 8 and 10-channel receivers of third-order directional acoustic sensors to achieve the desired directivity index.

A first-order directional acoustic sensor is also known as an Acoustic Vector Sensor (AVS). We have done a comparative study of various types of underwater AVS in the form of analytical, computer simulation, and experimental studies for the underwater environment (by drawing on previous research). Analytical approaches are utilised to compare the P-P, P-U, and PAGE-based AVS in Signal-to-Self Noise Ratio (SSNR) and SNR. We also derive an analytical expression for the SNR of particle velocity in the presence of correlated and uncorrelated ambient noise scenarios for various inter-element spacings.

We investigate the design parameters for a P-U based directional acoustic sensor using Finite Element Method (FEM) modelling and experimental work to acquire a more in-depth comprehension of its capabilities. We have built a prototype of AVS at the Indian Institute of Technology, Delhi and carried out experimental research at the National Institute of Ocean Technology (NIOT), Chennai. We have shown the beampattern in addition to the DoA estimation utilising a hydrophone array and the P-U based AVS.

A second-order directional acoustic sensor estimates pressure, particle velocity, and

particle velocity gradient along three orthogonal axes. FEM analysis demonstrates the influence of mutual coupling and investigates the effect of misalignment of the sensitive axis of accelerometers on the beampattern. By adjusting the intersensor distance between the accelerometers along the sensitive axis, we examine the variation in the directivity index. We have also developed the prototype of a second-order directional acoustic sensor and tested it in an underwater tank at NIOT Chennai. We also obtained the beampattern and Directivity Index (DI).

A third-order directional acoustic sensor estimates the pressure, particle velocity, particle velocity gradient, and second-order velocity gradient along three orthogonal axes. We have derived the DI of a third-order directional acoustic sensor along with the weights of different channels for optimum DI. The DI of a third-order directional acoustic sensor varies from 7.7 dB to 12 dB. We also test the DI by varying the inter-sensor distance between the accelerometers positioned along the axis. We have also done a sensor misalignment study and found that the beampattern of second and third-order directional acoustic sensors highly depends on it. We also present various methods that can be used to make a third-order directional acoustic sensor by using hydrophones and accelerometers.



## प्रस्तावना

विशेष दिशा में अन्य दिशाओं से आवाज़ को प्राथमिकता से डिटेक्ट करने वाले ध्वनिक ग्राहकों को दिशात्मक ध्वनिक ग्राहक कहा जाता है। थीसिस में कई उपकरणों के लिए निर्धारित ध्वनिक ग्राहकों के लिए अध्ययन किया गया है। विश्लेषणात्मक, सिमुलेशन और प्रायोगिक अध्ययन किए गए हैं, और प्रथम क्रम और द्वितीय क्रम के ध्वनिक ग्राहकों के प्रोटोटाइप विकसित किए गए हैं। साथ ही, तृतीय क्रम के ध्वनिक ग्राहकों के 8 और 10 चैनल रिसीवर के वेट्स का निर्धारण करने के लिए एक विश्लेषणात्मक संकेत भी निकाला गया है जो वांछित प्रत्यक्षता सूचकांक प्राप्त करने के लिए है।

प्रथम क्रम के ध्वनिक ग्राहक को एकांतर ध्वनिग्राही (एकूस) भी कहते हैं। हमने उपकरणों के तले पानी के वातावरण के लिए विभिन्न प्रकार के एकूस के तुलनात्मक अध्ययन किए हैं (पूर्व अनुसंधान पर आधारित)। सिग्नल-टू-सेल्फ नॉइज अनुपात (एसएसएनआर) और एसएनआर में पूर्वगणित विधियों के माध्यम से प्रेजर-प्रेजर, प्रेजर-यू और पेज-आधारित एकूस को तुलनात्मक विश्लेषण के लिए विश्लेषणात्मक दृष्टिकोण उपयोग किए गए हैं। हम अभिकंट तत्व विधि (एफईएम) मॉडलिंग का उपयोग करके प्रेस-यू आधारित ध्वनिक ग्राहक के डिज़ाइन पैरामीटर्स का अध्ययन करते हैं और इसकी क्षमता को और अधिगम करने के लिए प्रायोगिक काम किया है। हमने भारतीय प्रौद्योगिकी संस्थान, दिल्ली में एक एकूस के प्रोटोटाइप का निर्माण किया है और नेशनल इंस्टीट्यूट ऑफ़ ओसियन टेक्नोलॉजी (एनआईओटी), चेन्नई में प्रायोगिक अनुसंधान किया है। हमने हाइड्रोफोन एरे और प्रेस-यू आधारित एकूस का उपयोग करके बीमपैटर्न और डायरेक्टिविटी इंडेक्स (डीआई) का भी अनुमान लगाया है। द्वितीय क्रम के ध्वनिक ग्राहक तीन अपरोक्ष ध्वनि मापी, अणु वेग और अणु वेग ग्रेडिएंट को तीन अनुपातिक ध्वनि मापियों के माध्यम से आंकता करते हैं। एफईएम विश्लेषण द्वारा भौतिक संयोजन के प्रभाव की प्रदर्शनी और मानव बिगृधियों के असंबद्धता के प्रभाव की जाँच की गई है। संवेदकों के बीच अंतर-संवेदक दूरी को संवेदकों के संवेदनशील अक्ष के अलगाव से समीकरण करके, हम दृष्टिकोण इंडेक्स में परिवर्तन की जांच करते हैं। हमने द्वितीय क्रम के ध्वनिक ग्राहक के प्रोटोटाइप का भी विकास किया है और इसे नेशनल इंस्टीट्यूट ऑफ़ ओसियन टेक्नोलॉजी, चेन्नई में एक अधोदर टैंक में परीक्षण किया गया है। हमने बीमपैटर्न और डायरेक्टिविटी इंडेक्स (डीआई) की भी प्राप्ति की।

तृतीय क्रम के ध्वनिक ग्राहक में दबाव, अणु वेग, अणु वेग ग्रेडिएंट और द्वितीय क्रम के वेग ग्रेडिएंट का अनुमान लगाया जाता है जो तीन अनुपातिक ध्वनि मापियों के माध्यम से किया जाता है। हमने तृतीय क्रम के ध्वनिक ग्राहक

के डायरेक्टिविटी इंडेक्स (डीआई) को भी निकाला है साथ ही, विभिन्न चैनलों के वेट्स का निर्धारण करने के लिए विभिन्न तरीके प्रस्तुत किए हैं ताकि आवश्यक डीआई प्राप्त किया जा सके। तृतीय क्रम के ध्वनिक ग्राहक के डीआई विभिन्न अंतर-संवेदक दूरी को बदलकर भी टेस्ट किया गया है जो अक्ष के संवेदकों के अलगाव पर निर्भर करता है। हमने संवेदक असंबद्धता अध्ययन भी किया है और पाया है कि द्वितीय और तृतीय क्रम के ध्वनिक ग्राहक के बीमपैटर्न पर इसका अधिक निर्भर करता है। हम भी विभिन्न तरीकों को प्रस्तुत करते हैं जिनका उपयोग हाइड्रोफोन और एकूस का उपयोग करके तृतीय क्रम के ध्वनिक ग्राहक बनाने के लिए किया जा सकता है।

# Table of Contents

|                                                                                                            |              |
|------------------------------------------------------------------------------------------------------------|--------------|
| <b>Certificate</b>                                                                                         | <b>i</b>     |
| <b>Acknowledgements</b>                                                                                    | <b>ii</b>    |
| <b>Abstract</b>                                                                                            | <b>iii</b>   |
| <b>List of Figures</b>                                                                                     | <b>xii</b>   |
| <b>List of Tables</b>                                                                                      | <b>xxi</b>   |
| <b>Abbreviations</b>                                                                                       | <b>xxii</b>  |
| <b>List of Symbols</b>                                                                                     | <b>xxiii</b> |
| <b>1 Introduction</b>                                                                                      | <b>1</b>     |
| 1.1 Introduction . . . . .                                                                                 | 1            |
| 1.2 Theory of directional acoustic sensor . . . . .                                                        | 4            |
| 1.2.1 Sound . . . . .                                                                                      | 4            |
| 1.2.2 Euler’s conservation of momentum: relationship between pres-<br>sure and particle velocity . . . . . | 5            |
| 1.2.3 Acoustic impedance . . . . .                                                                         | 6            |
| 1.2.4 Reflection coefficient . . . . .                                                                     | 6            |
| 1.2.5 Acoustic intensity . . . . .                                                                         | 6            |
| 1.2.6 Coordinate system . . . . .                                                                          | 7            |
| 1.2.7 Arc-tangent method for DoA estimation . . . . .                                                      | 8            |
| 1.2.8 Directivity . . . . .                                                                                | 9            |

|          |                                                                                                 |           |
|----------|-------------------------------------------------------------------------------------------------|-----------|
| 1.3      | Taylor series expansion of an acoustic field . . . . .                                          | 10        |
| 1.4      | Literature survey . . . . .                                                                     | 11        |
| 1.5      | Modules constituting a directional acoustic sensor . . . . .                                    | 16        |
| 1.5.1    | Pressure sensor . . . . .                                                                       | 16        |
| 1.5.2    | Triaxial accelerometer . . . . .                                                                | 16        |
| 1.5.3    | Suspension system of accelerometer or geophone based direc-<br>tional acoustic sensor . . . . . | 16        |
| 1.6      | First-order directional acoustic sensor . . . . .                                               | 17        |
| 1.6.1    | Pressure sensor based first-order directional acoustic sensor . . .                             | 18        |
| 1.6.2    | Accelerometer based first-order directional acoustic sensor . . .                               | 20        |
| 1.7      | Highly directional acoustic vector sensor . . . . .                                             | 25        |
| 1.7.1    | Second-order directional acoustic sensor or Dyadic sensor . . . .                               | 26        |
| 1.7.2    | Design of second-order directional acoustic sensor or dyadic sensor                             | 26        |
| 1.7.3    | Third-order directional acoustic sensor . . . . .                                               | 28        |
| 1.7.4    | Design of third-order directional acoustic sensor . . . . .                                     | 30        |
| 1.8      | Motivation for the research work . . . . .                                                      | 31        |
| 1.9      | Key contributions . . . . .                                                                     | 32        |
| 1.10     | Outline of thesis . . . . .                                                                     | 33        |
| <b>2</b> | <b>Performance comparisons of P-P, PAGE and P-U based underwater<br/>AVS</b>                    | <b>35</b> |
| 2.1      | Introduction . . . . .                                                                          | 35        |
| 2.2      | SNR of particle velocity from pressure sensor based AVS . . . . .                               | 36        |
| 2.2.1    | Signal and noise received by P-P based AVS along an axis . . .                                  | 37        |
| 2.2.2    | Signal and noise received by PAGE based AVS along an axis .                                     | 41        |
| 2.3      | SNR of particle velocity using P-U based AVS . . . . .                                          | 43        |
| 2.3.1    | Signal and and noise received by P-U based AVS along an axis .                                  | 43        |
| 2.4      | Simulation set up and results . . . . .                                                         | 45        |
| 2.4.1    | Beampattern of P-P and PAGE based AVS as function of inter-<br>element spacing . . . . .        | 46        |

|          |                                                                                                                              |           |
|----------|------------------------------------------------------------------------------------------------------------------------------|-----------|
| 2.4.2    | Voltage response of different types of AVS and hydrophones . .                                                               | 48        |
| 2.4.3    | Absolute Angular Error (AAE) of PAGE based AVS due to mutual coupling of signals received by closely spaced pressure sensors | 49        |
| 2.5      | AVS experiment setup and results . . . . .                                                                                   | 51        |
| 2.5.1    | Experiment setup . . . . .                                                                                                   | 52        |
| 2.5.2    | Beampattern of P-P, PAGE and P-U based AVS at different frequencies . . . . .                                                | 54        |
| 2.5.3    | Voltage response of P-U and PAGE based AVS with different types of hydrophones . . . . .                                     | 56        |
| 2.6      | Chapter summary . . . . .                                                                                                    | 59        |
| <b>3</b> | <b>FEM simulation and experimental studies of P-U based AVS for design considerations</b>                                    | <b>61</b> |
| 3.1      | Introduction . . . . .                                                                                                       | 61        |
| 3.2      | FEM and experimental studies on the effect of a protective casing of P-U based AVS . . . . .                                 | 62        |
| 3.2.1    | FEM simulation setup . . . . .                                                                                               | 62        |
| 3.2.2    | Experimental study . . . . .                                                                                                 | 65        |
| 3.2.3    | Simulation and experiment results . . . . .                                                                                  | 69        |
| 3.3      | Analysis of Impact of viscoelastic rubber on energy coupled to AVS . .                                                       | 72        |
| 3.3.1    | Time of flight method for measuring sound speed . . . . .                                                                    | 72        |
| 3.3.2    | Measured speeds in various media . . . . .                                                                                   | 73        |
| 3.3.3    | Measurement of sound speed in viscoelastic rubber . . . . .                                                                  | 73        |
| 3.3.4    | Impact of viscoelastic rubber on energy coupled to AVS . . . . .                                                             | 76        |
| 3.4      | Vibration velocity of rigid sphere . . . . .                                                                                 | 77        |
| 3.5      | Experiment to find the beampattern of tri-axial accelerometer . . . . .                                                      | 79        |
| 3.6      | Development and experimental test of prototype P-U based AVS in underwater tank . . . . .                                    | 82        |
| 3.6.1    | Fabrication/Development . . . . .                                                                                            | 82        |
| 3.6.2    | Experimental studies of P-U based AVS at NIOT Chennai . . .                                                                  | 83        |

|          |                                                                                                                                    |            |
|----------|------------------------------------------------------------------------------------------------------------------------------------|------------|
| 3.7      | Field studies of P-U AVS based on experiment in Ganges river . . . . .                                                             | 86         |
| 3.7.1    | Experimental setup . . . . .                                                                                                       | 86         |
| 3.7.2    | Method . . . . .                                                                                                                   | 88         |
| 3.7.3    | Parameters used for experiment . . . . .                                                                                           | 89         |
| 3.7.4    | Results . . . . .                                                                                                                  | 89         |
| 3.8      | Chapter summary . . . . .                                                                                                          | 94         |
| <b>4</b> | <b>Simulation and experimental studies of second-order underwater directional sensor</b>                                           | <b>97</b>  |
| 4.1      | Introduction . . . . .                                                                                                             | 97         |
| 4.2      | Methodology . . . . .                                                                                                              | 98         |
| 4.3      | FEM studies of second-order directional sensor to study mutual coupling between the accelerometers placed along an axis . . . . .  | 100        |
| 4.3.1    | Simulation setup and results . . . . .                                                                                             | 100        |
| 4.4      | Simulation studies on sensitive axes of HDAVS due to combined effect of accelerometer placement translation and rotation . . . . . | 103        |
| 4.4.1    | Simulation setup and results . . . . .                                                                                             | 104        |
| 4.5      | Simulation studies for dependence of accelerometer separation on HDAVS DI. . . . .                                                 | 109        |
| 4.6      | Development of prototype HDAVS . . . . .                                                                                           | 110        |
| 4.6.1    | Design of HDAVS . . . . .                                                                                                          | 111        |
| 4.7      | Testing of HDAVS in underwater tank facility . . . . .                                                                             | 115        |
| 4.7.1    | Experiment setup . . . . .                                                                                                         | 115        |
| 4.8      | Chapter summary . . . . .                                                                                                          | 127        |
| <b>5</b> | <b>Analytical and simulation studies of third-order directional acoustic sensor</b>                                                | <b>130</b> |
| 5.1      | Methodology . . . . .                                                                                                              | 131        |
| 5.2      | Derivation of DI and channel weights of third-order directional sensor .                                                           | 134        |
| 5.2.1    | DI of third-order directional acoustic sensor consisting of 10-channel receiver . . . . .                                          | 134        |

|          |                                                                                                                                         |            |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------|------------|
| 5.2.2    | DI of third-order directional acoustic sensor consisting of 8-channel receiver . . . . .                                                | 138        |
| 5.3      | Simulation studies to determine the influence of accelerometer separation on DI of third-order directional acoustic sensor . . . . .    | 140        |
| 5.4      | FEM studies of the third-order directional acoustic sensor on the mutual coupling between accelerometers placed along an axis . . . . . | 143        |
| 5.4.1    | Simulation setup and results . . . . .                                                                                                  | 145        |
| 5.5      | Simulation studies on effect of misalignment of the sensitive axes . . . .                                                              | 147        |
| 5.5.1    | Simulation setup and results . . . . .                                                                                                  | 148        |
| 5.6      | Third-order directional acoustic sensor noise correlation in ambient noise                                                              | 156        |
| 5.7      | Methods for the design of third-order directional acoustic sensor along an axis . . . . .                                               | 157        |
| 5.7.1    | Developing a hydrophone based third-order directional acoustic sensor . . . . .                                                         | 159        |
| 5.7.2    | Developing a third-order directional acoustic sensor using an accelerometer and a hydrophone . . . . .                                  | 167        |
| 5.7.3    | Comparisons of all these approaches . . . . .                                                                                           | 180        |
| 5.7.4    | DI of third-order directional acoustic sensor obtained from experiment . . . . .                                                        | 183        |
| 5.8      | Chapter summary . . . . .                                                                                                               | 183        |
| <b>6</b> | <b>Conclusion and Scope for Future Research</b>                                                                                         | <b>187</b> |
| 6.1      | Scope for future research . . . . .                                                                                                     | 188        |
|          | <b>Bibliography</b>                                                                                                                     | <b>189</b> |
|          | <b>List of Publications</b>                                                                                                             | <b>204</b> |
|          | <b>Biography of Author</b>                                                                                                              | <b>205</b> |

# List of Figures

|      |                                                                                                                    |    |
|------|--------------------------------------------------------------------------------------------------------------------|----|
| 1.1  | Spherical coordinate system . . . . .                                                                              | 7  |
| 1.2  | P-P AVS in 2D using four hydrophones . . . . .                                                                     | 18 |
| 1.3  | Vertical cross-section of P-U AVS . . . . .                                                                        | 22 |
| 1.4  | The geometry of source and single triaxial accelerometer . . . . .                                                 | 23 |
| 1.5  | Second-order directional acoustic sensor . . . . .                                                                 | 26 |
| 1.6  | The geometry of source and dual uni-axial accelerometers . . . . .                                                 | 27 |
| 1.7  | The geometry of source and three accelerometers . . . . .                                                          | 29 |
| 1.8  | Third-order directional acoustic sensor . . . . .                                                                  | 29 |
| 2.1  | Beampattern of P-P based AVS for different inter-element spacing $d$ .                                             | 47 |
| 2.2  | Beampattern of PAGE based AVS for different inter-element spacing $d$                                              | 47 |
| 2.3  | Beampattern of P-U based AVS . . . . .                                                                             | 48 |
| 2.4  | Voltage response of P-P, PAGE and P-U AVS, and hydrophone . . . . .                                                | 50 |
| 2.5  | AAE in presence and absence of mutual coupling of signals received by<br>closely spaced pressure sensors . . . . . | 50 |
| 2.6  | Fabricated AVS . . . . .                                                                                           | 51 |
| 2.7  | Transmitted and received pulses . . . . .                                                                          | 53 |
| 2.8  | Beampatterns of P-P based AVS at different frequencies . . . . .                                                   | 54 |
| 2.9  | Beampattern of PAGE based AVS . . . . .                                                                            | 55 |
| 2.10 | Beampattern of P-U based AVS . . . . .                                                                             | 55 |
| 2.11 | Voltage gain of PAGE and PU based AVS . . . . .                                                                    | 56 |
| 2.12 | Voltage response of PU based AVS and B&K 8104 hydrophone . . . . .                                                 | 56 |
| 2.13 | Voltage response of PU based AVS and B&K 8103 hydrophone . . . . .                                                 | 57 |



|      |                                                                                                                       |    |
|------|-----------------------------------------------------------------------------------------------------------------------|----|
| 2.14 | Voltage response of PU based AVS and Benthowave-B-II 7003 hydrophone                                                  | 57 |
| 2.15 | Voltage response of PU based AVS and G.R.A.S 10-CT hydrophone                                                         | 58 |
| 3.1  | Model 1 geometry for finding the reflection coefficient between water and test material                               | 63 |
| 3.2  | Model 1 geometry meshed                                                                                               | 64 |
| 3.3  | Total acoustic pressure field in model 1 seen in FEM analysis                                                         | 65 |
| 3.4  | Model 2 geometry for finding the ratio of received energies between sensor with casing and without encasing           | 65 |
| 3.5  | Model 2 geometry meshed                                                                                               | 66 |
| 3.6  | Total acoustic pressure field in model 2                                                                              | 66 |
| 3.7  | Block diagram of experimental setup                                                                                   | 67 |
| 3.8  | Normalised response of underwater speaker at three different locations                                                | 67 |
| 3.9  | Hydrophone without protective casing                                                                                  | 68 |
| 3.10 | Experiment setup of hydrophone with hollow protective casing                                                          | 69 |
| 3.11 | Reflection coefficients of different materials for model 1 from FEM study                                             | 70 |
| 3.12 | Reflection coefficient at 1.5 kHz of polyurethane for model 1 on increasing thickness                                 | 70 |
| 3.13 | Ratio of received energies in dB (with casing/without casing) for different materials for model 2 in simulation study | 71 |
| 3.14 | Ratio of received energies in dB (with casing/without casing) for polyurethane on increasing thickness                | 71 |
| 3.15 | Ratio of received energies in dB (with casing/without casing) in tank experiment of Nylon material                    | 72 |
| 3.16 | Measurement of sound speed in brass                                                                                   | 73 |
| 3.17 | Measurement of sound speed in viscoelastic rubber                                                                     | 74 |
| 3.18 | Rubber disc placed at the centre of polyurethane foam                                                                 | 75 |
| 3.19 | Experiment setup for measuring sound speed in underwater tank                                                         | 75 |
| 3.20 | Incident plane wave and rigid sphere                                                                                  | 77 |

|      |                                                                                                                                                         |     |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 3.21 | $ka$ variation curves of amplitude of the ratio of vibration velocity of the rigid sphere and that of water particle . . . . .                          | 78  |
| 3.22 | $ka$ variation curves of amplitude of the ratio of vibration velocity of the rigid sphere made from different material and that of water particle . . . | 79  |
| 3.23 | $ka$ variation curves of phase ratio of vibration velocity of the rigid sphere and that of water particle . . . . .                                     | 80  |
| 3.24 | Experimental setup to find beampattern of triaxial accelerometer in anechoic chamber . . . . .                                                          | 80  |
| 3.25 | Directional response of accelerometer for $z$ -axis . . . . .                                                                                           | 81  |
| 3.26 | Directional response of accelerometer for $y$ -axis . . . . .                                                                                           | 81  |
| 3.27 | AVS prototype . . . . .                                                                                                                                 | 82  |
| 3.28 | Front view X-ray image of AVS prototype . . . . .                                                                                                       | 83  |
| 3.29 | Top view X-ray image of AVS prototype . . . . .                                                                                                         | 83  |
| 3.30 | Experimentation with P-U AVS at NIOT . . . . .                                                                                                          | 84  |
| 3.31 | Transmitted pulse train . . . . .                                                                                                                       | 85  |
| 3.32 | Transmitted and received pulses . . . . .                                                                                                               | 85  |
| 3.33 | Beampattern of $z$ channel of P-U AVS . . . . .                                                                                                         | 86  |
| 3.34 | Receive voltage sensitivity of $z$ channel of P-U AVS . . . . .                                                                                         | 87  |
| 3.35 | Setup on receiver boat . . . . .                                                                                                                        | 88  |
| 3.36 | Hydrophone noise . . . . .                                                                                                                              | 90  |
| 3.37 | AVS noise along with mounting frame vibrations . . . . .                                                                                                | 90  |
| 3.38 | Signal plus noise received using hydrophones . . . . .                                                                                                  | 91  |
| 3.39 | Spectrogram of hydrophone signals . . . . .                                                                                                             | 92  |
| 3.40 | Signal plus noise received using AVS . . . . .                                                                                                          | 92  |
| 3.41 | Spectrogram of AVS signals . . . . .                                                                                                                    | 93  |
| 3.42 | PSD of received signal corrupted by noise . . . . .                                                                                                     | 93  |
| 3.43 | Comparison of DoA estimates (AVS vs hydrophone array) . . . . .                                                                                         | 94  |
| 4.1  | Multiple accelerometers based HDAVS . . . . .                                                                                                           | 99  |
| 4.2  | Cross-section of HDAVS simulation setup . . . . .                                                                                                       | 101 |
| 4.3  | Meshing of HDAVS setup . . . . .                                                                                                                        | 101 |

|      |                                                                                                                                                                                              |     |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 4.4  | Total acoustic pressure field of HDAVS setup . . . . .                                                                                                                                       | 102 |
| 4.5  | Estimated beampattern of HDAVS . . . . .                                                                                                                                                     | 103 |
| 4.6  | AE of estimated beampattern of HDAVS . . . . .                                                                                                                                               | 103 |
| 4.7  | True value and misaligned placement simulation . . . . .                                                                                                                                     | 104 |
| 4.8  | Beampattern of HDAVS along $x$ -axis using velocity gradient for different $d$                                                                                                               | 105 |
| 4.9  | $RMSE$ of HDAVS along $x$ -axis using velocity gradient for different $d$ .                                                                                                                  | 106 |
| 4.10 | Beampattern of HDAVS along $x$ -axis using velocity gradient for different $\theta_1$ . . . . .                                                                                              | 107 |
| 4.11 | $RMSE$ of HDAVS along $x$ -axis using velocity gradient for different $\theta_1$ .                                                                                                           | 107 |
| 4.12 | Beampattern of HDAVS along $x$ -axis using velocity gradient for $(\theta_1 = 5^\circ, \phi_1 = 95^\circ)$ and $(\theta_2 = 355^\circ, \phi_2 = 85^\circ)$ . . . . .                         | 108 |
| 4.13 | $AE$ of HDAVS along $x$ -axis using velocity gradient for $(\theta_1 = 5^\circ, \phi_1 = 95^\circ)$ and $(\theta_2 = 355^\circ, \phi_2 = 85^\circ)$ . . . . .                                | 108 |
| 4.14 | HDAVS (5 channel receiver) DI for various separation distance $d$ . . . .                                                                                                                    | 110 |
| 4.15 | Difference in HDAVS (5 channel receiver) DI for various separation distance $d$ from ideal DI at $\theta = 0^\circ$ . . . . .                                                                | 111 |
| 4.16 | HDAVS housing design with three removable sections . . . . .                                                                                                                                 | 111 |
| 4.17 | HDAVS casing 3D printed according to the aforementioned design . . .                                                                                                                         | 112 |
| 4.18 | The design of the HDAVS sensor includes one tri-axial accelerometer and four uni-axial accelerometers, all of which are housed inside the exterior casing at predetermined distance. . . . . | 113 |
| 4.19 | (a) Designed 3D Model for mold. (b) 3D printed model from Dept. of Design IIT, Delhi . . . . .                                                                                               | 114 |
| 4.20 | (a) Placement of mold kept for solidification for 24 hours. (b) Mold removed and desired depressions obtained for placement of accelerometers                                                | 114 |
| 4.21 | Lower section of the developed HDAVS . . . . .                                                                                                                                               | 114 |
| 4.22 | Developed HDAVS. . . . .                                                                                                                                                                     | 115 |
| 4.23 | X-ray images of the constructed HDAVS taken from the side and the top in order to check for alignment. . . . .                                                                               | 116 |

|      |                                                                                                                                              |     |
|------|----------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 4.24 | The underwater tank dimensions, the positions of the source and suspended HDAVS are shown. . . . .                                           | 116 |
| 4.25 | Testing setup at Acoustic Test facility, NIOT, Chennai. HDAVS and transmitter are shown. . . . .                                             | 117 |
| 4.26 | Orientation of the setup of 5 accelerometers and their sensitive axes along with representation of transducer emitting sound waves . . . . . | 117 |
| 4.27 | Travel path of signal inside tank . . . . .                                                                                                  | 118 |
| 4.28 | Reflection free window for 2ms duration signal in tank . . . . .                                                                             | 118 |
| 4.29 | Snapshot of data captured for one channel of an accelerometer . . . . .                                                                      | 119 |
| 4.30 | Beampattern of accelerometers $A0$ and $A3$ at 2 kHz along $x$ -axis. . . . .                                                                | 119 |
| 4.31 | AE of beampattern of $A0$ - $X$ for 2 kHz tonal from a distance of 10.8 m. . . . .                                                           | 120 |
| 4.32 | AE of beampattern of $A3$ - $X$ for 2 kHz tonal from a distance of 10.8 m. . . . .                                                           | 120 |
| 4.33 | Beampattern of accelerometers $A0$ and $A1$ at 2 kHz along $y$ -axis. . . . .                                                                | 121 |
| 4.34 | AE of beampattern of $A0$ - $Y$ for 2 kHz tonal from a distance of 10.8 m. . . . .                                                           | 121 |
| 4.35 | AE of beampattern of $A1$ - $Y$ for 2 kHz tonal from a distance of 10.8 m. . . . .                                                           | 122 |
| 4.36 | Beampattern along $x$ -axis for 2 kHz tonal from a distance of 10.8 m. . . . .                                                               | 122 |
| 4.37 | Beampattern along $y$ -axis for 2 kHz tonal from a distance of 10.8 m. . . . .                                                               | 123 |
| 4.38 | Beampattern along $x$ -axis for 1.5 kHz tonal from a distance of 10.8 m. . . . .                                                             | 123 |
| 4.39 | Beampattern along $y$ -axis for 1.5 kHz tonal from a distance of 10.8 m. . . . .                                                             | 124 |
| 4.40 | AE of beampattern of velocity gradient along $x$ -axis for 2 kHz tonal from a distance of 10.8 m. . . . .                                    | 124 |
| 4.41 | AE of beampattern of velocity gradient along $y$ -axis for 2 kHz tonal from a distance of 10.8 m. . . . .                                    | 125 |
| 4.42 | AE of beampattern of velocity gradient along $x$ -axis for 1.5 kHz tonal from a distance of 10.8 m. . . . .                                  | 125 |
| 4.43 | AE of beampattern of velocity gradient along $y$ -axis for 1.5 kHz tonal from a distance of 10.8 m. . . . .                                  | 126 |
| 4.44 | Estimated DI of second-order directional sensor consisting of 6 channel receiver at 2 kHz . . . . .                                          | 126 |

|      |                                                                                                                                                             |     |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 5.1  | Beampattern of different channel of third-order directional acoustic sensor . . . . .                                                                       | 133 |
| 5.2  | Directivity index of 10 component receiver . . . . .                                                                                                        | 138 |
| 5.3  | Directivity index of 8 component receiver . . . . .                                                                                                         | 140 |
| 5.4  | Beampattern of second-order velocity gradient along $x$ axis for different inter-sensor distance . . . . .                                                  | 141 |
| 5.5  | 10-channel receiver DI for various separation distance $d$ . . . . .                                                                                        | 142 |
| 5.6  | 8-channel receiver DI for various separation distance $d$ . . . . .                                                                                         | 143 |
| 5.7  | Difference in DI of third-order directional acoustic sensor(8-channel receiver) for various separation distance $d$ from ideal DI at $\theta = 0^0$ . . . . | 144 |
| 5.8  | Difference in DI of third-order directional acoustic sensor(10-channel receiver) for various separation distance $d$ from ideal DI at $\theta = 0^0$ . . .  | 144 |
| 5.9  | Cross-section of third-order directional acoustic sensor simulation setup                                                                                   | 145 |
| 5.10 | Meshing of third-order directional acoustic sensor setup . . . . .                                                                                          | 146 |
| 5.11 | Total acoustic pressure field of third-order directional acoustic sensor setup . . . . .                                                                    | 146 |
| 5.12 | Estimated beampattern of third-order directional acoustic sensor along an axis . . . . .                                                                    | 147 |
| 5.13 | AE of estimated beampattern of third-order directional acoustic sensor along an axis . . . . .                                                              | 147 |
| 5.14 | Multiple accelerometers based third-order directional sensor . . . . .                                                                                      | 148 |
| 5.15 | True value and misaligned placement simulation . . . . .                                                                                                    | 149 |
| 5.16 | Beampattern of third-order directional acoustic sensor using second-order velocity gradient for different $d$ . . . . .                                     | 151 |
| 5.17 | $RMSE$ of third-order directional acoustic sensor using second-order velocity gradient for different $d$ . . . . .                                          | 151 |
| 5.18 | Beampattern of third-order directional acoustic sensor using second-order velocity gradient for different $\theta_1$ . . . . .                              | 152 |
| 5.19 | $RMSE$ of third-order directional acoustic sensor using second-order velocity gradient for different $\theta_1$ . . . . .                                   | 152 |

|      |                                                                                                                                                                                                                                 |     |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 5.20 | Beampattern of third-order directional acoustic sensor using second-order velocity gradient for $(\theta_1 = 2.5^\circ, \phi_1 = 92.5^\circ), (\theta_0 = -2.5^\circ, \phi_0 = 87.5^\circ)$                                     | 153 |
| 5.21 | <i>AE</i> of third-order directional acoustic sensor using second-order velocity gradient for $(\theta_1 = 2.5^\circ, \phi_1 = 92.5^\circ), (\theta_0 = 357.5^\circ, \phi_0 = 87.5^\circ)$                                      | 153 |
| 5.22 | Beampattern of third-order directional acoustic sensor using gradient for $(\theta_1 = 5^\circ, \phi_1 = 95^\circ), (\theta_0 = 7.5^\circ, \phi_0 = 97.5^\circ), (\theta_2 = -5^\circ, \phi_2 = 85^\circ)$                      | 154 |
| 5.23 | <i>AE</i> of third-order directional acoustic sensor using second-order velocity gradient for $(\theta_1 = 5^\circ, \phi_1 = 95^\circ), (\theta_0 = 7.5^\circ, \phi_0 = 97.5^\circ), (\theta_2 = 355^\circ, \phi_2 = 85^\circ)$ | 154 |
| 5.24 | Beampattern of third-order directional acoustic sensor using second-order velocity gradient for $d_1 = \lambda/8, d_2 = \lambda/16$                                                                                             | 155 |
| 5.25 | <i>AE</i> of third-order directional acoustic sensor using second-order velocity gradient for $d_1 = \lambda/8, d_2 = \lambda/16$                                                                                               | 155 |
| 5.26 | Two hydrophone based third-order directional sensor                                                                                                                                                                             | 159 |
| 5.27 | Particle velocity beampattern along $x$ direction using two hydrophones                                                                                                                                                         | 160 |
| 5.28 | Velocity gradient beampattern along $x$ direction using two hydrophones                                                                                                                                                         | 161 |
| 5.29 | Second-order velocity gradient beampattern along $x$ direction using two hydrophones                                                                                                                                            | 161 |
| 5.30 | Estimated DI of third-order directional acoustic sensor consists of 8-channel receiver using two hydrophones                                                                                                                    | 162 |
| 5.31 | Estimated DI of third-order directional acoustic sensor consists of 10-channel receiver using two hydrophones                                                                                                                   | 162 |
| 5.32 | Three hydrophone based third-order directional sensor                                                                                                                                                                           | 163 |
| 5.33 | Particle velocity beampattern along $x$ direction using three hydrophones                                                                                                                                                       | 165 |
| 5.34 | Velocity gradient beampattern along $x$ direction using three hydrophones                                                                                                                                                       | 165 |
| 5.35 | Second-order velocity beampattern gradient along $x$ direction using three hydrophones                                                                                                                                          | 166 |
| 5.36 | Estimated DI of third-order directional acoustic sensor consists of 8-channel receiver using three hydrophones                                                                                                                  | 166 |

|      |                                                                                                                                              |     |
|------|----------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 5.37 | Estimated DI of third-order directional acoustic sensor consists of 10-channel receiver using three hydrophones . . . . .                    | 167 |
| 5.38 | Single hydrophone and a dual-axis accelerometer for third-order directional acoustic sensor . . . . .                                        | 168 |
| 5.39 | Particle velocity beampattern along $x$ direction using dual-axis accelerometer . . . . .                                                    | 169 |
| 5.40 | Velocity gradient beampattern along $x$ direction using dual-axis accelerometer . . . . .                                                    | 169 |
| 5.41 | Second-order velocity gradient beampattern along $x$ direction using dual-axis accelerometer . . . . .                                       | 170 |
| 5.42 | Estimated DI of third-order directional acoustic sensor consists of 8-channel receiver using one accelerometer and a hydrophone . . . . .    | 170 |
| 5.43 | Estimated DI of third-order directional acoustic sensor consists of 10-channel receiver using three accelerometer and a hydrophone . . . . . | 171 |
| 5.44 | Combining two accelerometers and one hydrophone for third-order directional acoustic sensor . . . . .                                        | 171 |
| 5.45 | Particle velocity beampattern along $x$ direction using two uni-axial accelerometers . . . . .                                               | 173 |
| 5.46 | Velocity gradient beampattern along $x$ direction using two uni-axial accelerometers . . . . .                                               | 173 |
| 5.47 | Second-order velocity gradient beampattern along $x$ direction using two uni-axial accelerometers . . . . .                                  | 174 |
| 5.48 | Estimated DI of third-order directional acoustic sensor consisting of 8-channel receiver using two accelerometers . . . . .                  | 174 |
| 5.49 | Estimated DI of third-order directional acoustic sensor consisting of 10-channel receiver using two accelerometers . . . . .                 | 175 |
| 5.50 | Combining three accelerometers and one hydrophone for third-order directional acoustic sensor . . . . .                                      | 175 |
| 5.51 | Particle velocity beampattern along $x$ direction using three uni-axial accelerometers . . . . .                                             | 176 |

|      |                                                                                                                                                  |     |
|------|--------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| 5.52 | Velocity gradient beampattern along $x$ direction using three uni-axial accelerometers . . . . .                                                 | 176 |
| 5.53 | Second-order velocity gradient beampattern along $x$ direction using three uni-axial accelerometers . . . . .                                    | 177 |
| 5.54 | Estimated DI of third-order directional acoustic sensor consists of 8-channel receiver using three accelerometers . . . . .                      | 177 |
| 5.55 | Estimated DI of third-order directional acoustic sensor consists of 10-channel receiver using three accelerometers . . . . .                     | 178 |
| 5.56 | RMSE of third-order directional sensor using two and three hydrophones                                                                           | 179 |
| 5.57 | RMSE of third-order directional sensor using one and two accelerometers                                                                          | 180 |
| 5.58 | RMSE of third-order directional sensor using two and three accelerometers . . . . .                                                              | 181 |
| 5.59 | Estimated DI from experiments of third-order directional acoustic sensor consists of 8-channel receiver at 2 kHz using dual axis accelerometers  | 182 |
| 5.60 | Estimated DI from experiments of third-order directional acoustic sensor consists of 10-channel receiver at 2 kHz using dual axial accelerometer | 182 |



# List of Tables

|     |                                                                                           |     |
|-----|-------------------------------------------------------------------------------------------|-----|
| 2.1 | Noise power added to the single channel of P-P and P-U AVS. . . . .                       | 45  |
| 2.2 | Simulation parameters and values. . . . .                                                 | 46  |
| 2.3 | Hydrophones sensitivity in $dBre1V/\mu Pa$ from data sheets . . . . .                     | 49  |
| 2.4 | Voltage response of P-U based AVS and some commercially available<br>hydrophones. . . . . | 49  |
| 2.5 | Experiment parameters and their values. . . . .                                           | 52  |
| 2.6 | Comparison of P-P, PAGE and P-U based AVS . . . . .                                       | 58  |
| 3.1 | Material properties. . . . .                                                              | 67  |
| 3.2 | Speed of sound as measured experimentally . . . . .                                       | 74  |
| 3.3 | Parameter used for experiment . . . . .                                                   | 89  |
| 4.1 | Cases for misalignment study for HDAVS directional sensor . . . . .                       | 105 |
| 4.2 | Ideal and estimated HPBW of first-order and second-order directional<br>sensor . . . . .  | 123 |
| 5.1 | Cases for misalignment study for third-order directional acoustic sensor                  | 150 |
| 5.2 | HPBW of cosine, cosine-square and cosine-cube dipole . . . . .                            | 156 |
| 5.3 | Correlation between sensors . . . . .                                                     | 158 |
| 5.4 | Parameters used for experiment . . . . .                                                  | 158 |

# Abbreviations

|        |                                                  |
|--------|--------------------------------------------------|
| AVS    | Acoustic Vector Sensor                           |
| PAGE   | Phase and Amplitude Gradient                     |
| HDAVS  | Highly directional Acoustic Vector Sensor        |
| FEM    | Finite Element Method                            |
| AE     | Absolute Error                                   |
| AAE    | Absolute Angular Error                           |
| IBP    | Ideal Beampattern                                |
| EBP    | Estimated Beampattern                            |
| RMSE   | Root Mean Square Error                           |
| NIOT   | National Institute of Ocean Technology           |
| HPBW   | Half Power Beamwidth                             |
| DIFAR  | Directional Frequency Analysis and Recording     |
| DoA    | Direction of Arrival                             |
| MSAE   | Mean Square Angular Error                        |
| SNR    | Signal to Noise Ratio                            |
| SSNR   | Signal to Self Noise Ratio                       |
| MF     | Microflow                                        |
| WiMUST | Widely Scaled Mobile Underwater Sonar Technology |
| DAVS   | Dual Accelerometer Vector Sensor                 |
| PML    | Perfectly Matched Layer                          |
| PDF    | Probability Density Function                     |
| DI     | Directivity Index                                |

# List of Symbols

The next list describes several symbols that will be later used within the body of the document

$B(\theta, \phi)$  Beampattern

$\alpha$  Absorption coefficient

$\beta$  Specific gravity

$\mathbf{a}(t)$  Acceleration vector in time domain

$\mathbf{I}(t)$  Intensity vector in time domain

$\mathbf{r}$  Position vector

$\mathbf{u}(t)$  Velocity vector in time domain

$\mathbf{U}$  Velocity vector in frequency domain

$\Gamma$  Reflection coefficient

$\lambda$  Wavelength

$\nabla P(\omega)$  Amplitude of pressure gradient

$\nabla \Phi$  Phase gradient

$\omega$  Radian frequency

$\phi$  Declination angle

|                         |                                                     |
|-------------------------|-----------------------------------------------------|
| $\rho$                  | Medium density                                      |
| $\sigma_{AN}^2(\omega)$ | Power spectral density of of ambient noise          |
| $\sigma_{SN}^2(\omega)$ | Power of spectral density of self noise             |
| $\sigma_{TN}^2(\omega)$ | Power spectral density of total noise               |
| $\tau$                  | Time delay                                          |
| $\theta$                | Azimuth angle                                       |
| $a(\theta, \phi)$       | Directional cosine of along x-axis                  |
| $a_1(\theta, \phi)$     | Approximated directional cosine along x-axis        |
| $a_2(\theta, \phi)$     | Approximated directional cosine-square along x-axis |
| $a_3(\theta, \phi)$     | Approximated directional cosine-cube along x-axis   |
| $b(\theta, \phi)$       | Directional cosine along y-axis                     |
| $b_1(\theta, \phi)$     | Approximated directional cosine along y-axis        |
| $b_2(\theta, \phi)$     | Approximated directional cosine-square along y-axis |
| $b_3(\theta, \phi)$     | Approximated directional cosine-cube along y-axis   |
| $c(\theta, \phi)$       | Directional cosine along z-axis                     |
| $c$                     | Speed of sound                                      |
| $c_1(\theta, \phi)$     | Approximated directional cosine along z-axis        |
| $c_2(\theta, \phi)$     | Approximated directional cosine-square along z-axis |
| $c_3(\theta, \phi)$     | Approximated directional cosine-cube along z-axis   |
| $d$                     | Inter-element spacing                               |
| $E_{coupled}$           | Energy of coupled signal                            |

|                |                                                                                                          |
|----------------|----------------------------------------------------------------------------------------------------------|
| $E_{Incident}$ | Incident signal energy                                                                                   |
| $k$            | Wave number                                                                                              |
| $n(t)$         | Noise                                                                                                    |
| $p(t)$         | Acoustic pressure in time domain                                                                         |
| $p_i(t)$       | Incident pressure wave in time domain                                                                    |
| $p_r(t)$       | Reflected pressure wave in time domain                                                                   |
| $S_a$          | Sensitivity factor of accelerometer in $mv/g$                                                            |
| $S_P$          | Sensitivity factor of pressure sensor in $dBre1V \mu Pa$                                                 |
| $U_0(\omega)$  | Particle velocity along x-axis in frequency domain                                                       |
| $u_0(t)$       | Particle velocity along x-axis in time domain                                                            |
| $U_1(\omega)$  | Velocity gradient along x-axis in frequency domain                                                       |
| $u_1(t)$       | Velocity gradient along x-axis in time domain                                                            |
| $U_2(\omega)$  | Gradient of $U_1(\omega)$ along x-axis in frequency domain                                               |
| $u_2(t)$       | Gradient of $u_1(t)$ along x-axis in time domain                                                         |
| $U_i(\omega)$  | $i^{th}$ partial derivative of particle velocity along $x$ -axis with respect to $x$ in frequency domain |
| $u_i(t)$       | $i^{th}$ partial derivative of particle velocity along $x$ -axis with respect to $x$ in time domain      |
| $V_0(\omega)$  | Particle velocity along y-axis in frequency domain                                                       |
| $v_0(t)$       | Particle velocity along y-axis in time domain                                                            |
| $V_1(\omega)$  | Velocity gradient along y-axis in frequency domain                                                       |
| $v_1(t)$       | Velocity gradient along y-axis in time domain                                                            |

|               |                                                                                                          |
|---------------|----------------------------------------------------------------------------------------------------------|
| $V_2(\omega)$ | Gradient of $V_1(\omega)$ y-axis in frequency domain                                                     |
| $v_2(t)$      | Gradient of $v_1(t)$ along y-axis in time domain                                                         |
| $V_i(\omega)$ | $i^{th}$ partial derivative of particle velocity along $y$ -axis with respect to $y$ in frequency domain |
| $v_i(t)$      | $i^{th}$ partial derivative of particle velocity along $y$ -axis with respect to $y$ in time domain      |
| $W_0(\omega)$ | Particle velocity along z-axis in frequency domain                                                       |
| $w_0(t)$      | Particle velocity along z-axis in time domain                                                            |
| $W_1(\omega)$ | Velocity gradient along z-axis in frequency domain                                                       |
| $w_1(t)$      | Velocity gradient along z-axis in time domain                                                            |
| $W_2(\omega)$ | Gradient of $W_1(\omega)$ z-axis in frequency domain                                                     |
| $w_2(t)$      | Gradient of $w_1(t)$ along z-axis in time domain                                                         |
| $W_i(\omega)$ | $i^{th}$ partial derivative of particle velocity along $z$ -axis with respect to $z$ in frequency domain |
| $w_i(t)$      | $i^{th}$ partial derivative of particle velocity along $z$ -axis with respect to $z$ in time domain      |
| $y_c(t)$      | Particle velocity along y-axis in time domain                                                            |
| $y_o(t)$      | Pressure signal in time domain                                                                           |
| $y_s(t)$      | Particle velocity along z-axis in time domain                                                            |
| $z$           | Acoustic impedance                                                                                       |