VARIATIONAL INEQUALITY AND ITS VARIANTS WITH APPLICATIONS TO EQUILIBRIUM PROBLEMS

RACHANA GUPTA

DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY DELHI
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by

RACHANA GUPTA

Department of Mathematics

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Certificate

This is to certify that the thesis entitled **"Variational Inequality and Its Variants with Applications to Equilibrium Problems"** submitted by **"Ms. Rachana Gupta"** to the Indian Institute of Technology Delhi, for the award of the Degree of **Doctor of Philosophy**, is a record of the original bona fide research work carried out by her under my supervision and guidance. The thesis has reached the standards fulfilling the requirements of the regulations relating to the degree.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

New Delhi
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Dr. Aparna Mehra
Associate Professor
Department of Mathematics
Indian Institute of Technology Delhi

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Abstract

Modeling, analysis and computation of different equilibria using mathematical programming techniques is cited as the "Equilibrium programming". There are numerous efficient methods that had been developed to analyze and numerically solve models of economic and game theoretic equilibrium in recent years. The initial methods which were used to compute economic equilibria, were fixed point methods which were proved to be theoretically very powerful. Another approach for solving equilibrium models is via nonlinear optimization. Since this approach requires very restrictive assumptions on the models in order to work, it does not provide a satisfactory alternative to the fixed point methods. Prompted by this experience, came a move of research in iterative methods for solving variational inequalities and nonlinear complimentarity problems which arise often from or in several equilibrium contexts. The finite dimensional variational inequalities, which is a generalization of nonlinear complimentarity problems provided a broad unifying setting for the study of optimization and equilibrium problems and serves as the main computational framework for the practical solution of problems in mathematical science. Large number of equilibrium problems arising in economics and transportation science can be studied in the general framework of different variational inequalities.

There are various ways to approach a variational inequality problem. One of the principal ways is to devise an optimization problem whose solution would lead to the solution of the original variational inequality problem. This is done by introducing the notion of a gap function which also plays an important role in the study and analysis of error bounds for variational inequality problems. An error bound is an expression which provides an upper estimate of the distance of an arbitrary feasible point to the solution set of the variational inequality problem. If one can reformulate the variational inequality

problem as an optimization problem then it is easy to solve the variational inequality problem using efficient algorithms for corresponding optimization problems.

The thesis concentrates on constructing new gap functions and developing error bounds for various kind of variational inequality problems including quasi variational inequality problem and inverse quasi variational inequality problem. Towards the end, we also reformulated the time dependent generalized Nash equilibrium problem into one of the evolutionary variational inequality problem.

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