

VIBRATIONS OF UNSYMMETRICAL SANDWICH
STRUCTURES

A Thesis submitted in fulfilment of
the requirements for the Degree of
DOCTOR OF PHILOSOPHY

by

Y.V.K. Sadasiva Rao

to the

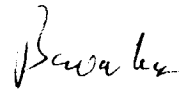
Department of Mechanical Engineering
Indian Institute of Technology, New Delhi.

MARCH 1972

CERTIFICATE

This is to certify that the Thesis entitled "Vibrations of unsymmetrical Sandwich Structures" being submitted by Mr. Y.V.K. Sadasiva Rao to the Indian Institute of Technology, Delhi, for the award of the Degree of Doctor of Philosophy in Mechanical Engineering, is record of bonafide research work carried out by him. He has worked under my guidance and has fulfilled the requirements for the submission of this Thesis, which has reached the requisite standard.

The results contained in this Thesis have not been submitted in part or in full, to any other University or Institute for the award of any degree or diploma.

 24.3.72
(B.C. NAKRA)
Professor,
Mechanical Engineering Deptt.
Indian Institute of Technology,
New Delhi.

ACKNOWLEDGEMENTS

It is with great pleasure the author records his deep sense of gratitude to Dr. B.C. Nakra who suggested the problem and supervised the work on this Thesis. Dr. Nakra's unceasing enthusiasm, inspiring suggestions and constant encouragement greatly helped the author at all stages of work.

The author expresses his sincere appreciations of the help rendered by the staff of the Computer Centre, Indian Institute of Technology, Delhi. The experimental work concerning the Thesis was carried out with the assistance of the staff of the Vibrations, Control and Instrumentation Laboratory which the author gratefully acknowledges.

Thanks are also due to author's numerous friends and colleagues for their painstaking help in the preparation of the final copies of the Thesis.

A special word of thanks is to Mr. Ranjit Kumar who typed the manuscript.

ABSTRACT

The analysis of flexural vibrations of unsymmetrical sandwich beams has been carried out by deriving frequency equation from equations of motion in which the membrane energy of the faces, energy due to bending of the faces about their own axes, shear and extensional energy of the core, transverse, longitudinal translatory and rotary inertia have been included. For the case of free vibrations, a series solution is assumed to satisfy simply supported conditions, and the non-dimensional frequency parameter has been evaluated for all the three resulting families of modes, thus obtained. The influence of rotary and longitudinal translatory inertia is investigated by evaluating the frequency parameters for various values of modal numbers, face thickness ratio, ratio of core thickness to face thickness, ratio of shear modulus of core to Young's modulus of face material and ratio of thickness of face to length of the beam. Influence of these terms is also studied on the strain response in case of sandwich beam with viscoelastic core, subjected to harmonic excitation. Damping action due to strains in viscoelastic layers is investigated by finding the value of system loss factor for varying modal number and other

non-dimensional ratios.

Equilibrium equations and boundary conditions have been derived for the case of unsymmetrical sandwich plate by taking membrane energy of the faces, energy due to bending of the faces about their own axes, shear energy of the core, transverse, longitudinal translatory and rotary inertia and making use of Hamilton's principle. Series solution is assumed for simply supported boundary conditions and the influence of rotatory and longitudinal translatory inertia, for the case of free vibrations is investigated for various non-dimensional parameters. Vibration response is obtained for the case when the core is assumed to be viscoelastic and when the plate is under uniform excitation. Expressions defining damping at various modes of vibration have also been given.

Further work has been restricted to the vibration analysis of unsymmetrical sandwich plate with elastic core for the two types of edge conditions the first type with all edges clamped and second type with two edges simply supported and the remaining two clamped, by using Raleigh-Ritz method. The values of frequency parameter have been given for varying non-dimen-

sional parameters including modal numbers in both cases. Experimental work has been aimed for comparison with the theoretically obtained results on a few samples of laminated plate in both the cases.

Contents

	<u>Page</u>
CERTIFICATE	
ACKNOWLEDGEMENTS	
ABSTRACT	
CONTENTS	
LIST OF FIGURES	
LIST OF TABLES	
TERMINOLOGY	
 <u>CHAPTER I</u>	 1
IA : Introduction	1
IB : Literature Survey	5
IC : Statement and Scope of the Problem	18
 <u>CHAPTER II</u> UNSYMMETRICAL SANDWICH BEAM - FORMULATION AND SOLUTION	 23
II.A : Introduction	23
II.B : Assumptions	24
II.C : Formulation	24
1 : Equations of Motion-Derivation	24
2 : Solution of Simply Supported Beam with Elastic Core	29
3 : Influence of Rotary and Translatory Inertia on the Frequencies of Different Modes of Vibration	37

<u>CHAPTER II (contd.)</u>	<u>PAGE</u>
II.D : Forced Vibrations of Simply Supported Beam with Viscoelastic Core	66
1 : Solution	66
2 : Discussion of the results	72
II.E : Damping of Sandwich Beam with Visco- elastic Core	76
1 : Expressions Defining Damping	76
2 : Discussion	78
CHAPTER III : UNSYMMETRICAL SANDWICH PLATE - DERIVATION OF EQUATIONS OF MOTION	84
III.A : Introduction to Procedure Employed	84
III.B Assumptions	85
III.C Derivation of Equations of Motion	87;
CHAPTER IV : SOLUTION FOR SIMPLY SUPPORTED UNSYMMETRICAL SANDWICH PLATE	100
IV.A.1 : Solution for Elastic Core	100
2 : Influence of Rotary and Translatory Inertia as the Frequencies of different modes of Vibration	113
IV.B. Forced Vibrations with Viscoelastic Core	127
1 : Expression of the Results	133
IV.C : Damping with Viscoelastic Core	136
1 : Expressions Defining Damping	136

CHAPTER V : UNSYMMETRICAL SANDWICH PLATE-DERIVATION AND SOLUTION FOR ALL CLAMPED EDGES	137
V.A Introduction to Procedure Employed	137
V.B Solution	138
1. Solution with Elastic Core	138
2. Simplified Case	154
3. Discussion	157
V.C Experimental determination of Frequencies of Clamped Sandwich Plate	166
1. Description of set up	166
2. Preparation of specimen	167
3. Testing Procedure	168
4. Comparison of theoretical & exptl. results	170
CHAPTER VI : UNSYMMETRICAL SANDWICH PLATE-DERIVATION AND SOLUTION FOR UNSYMMETRICAL BOUNDARY CONDITIONS	173
VI.A Introduction to Procedure Employed	173
VI.B.1 Solution with Elastic Core	173
2 Discussion	183
VI.C Experimental determination of the frequencies of sandwich plate with two opposite sides simply supported and the remaining two clamped	192
1 Comparison of theoretical and experimental results	194
CHAPTER VII	196
VII.A General Discussions and Conclusions	196
VII.B Further Work	204

	<u>Page</u>
REFERENCES	206
APPENDIX A : Details of Derivation in III.C	220
APPENDIX B : Details of Derivation in V.C.1	230
APPENDIX C : Details of Derivation in VI.C.1	236

...