ON CERTAIN NON-ASSOCIATIVE RINGS AND ALGEBRAS WITH COMMUTATORS IN THE NUCLEI

By

S. SHARADHA

A THESIS SUBMITTED TO THE
INDIAN INSTITUTE OF TECHNOLOGY, DELHI
FOR THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY

Dep rtm nt of M thematics
INDIAN INSTITUTE OF TECHNOLOGY
HAUZ KHAS, NEW DELHI-110 016

1986

:CERTIFICATE:

This is to certify that the thesis entitled "On certian non-associative rings and algebras with commutators in the nuclei" being submitted by Ms Sharadha to the Indian Institute of Technology, Delhi, for the award of Ph.D. degree in Mathematics, is a record of bonafide research work carried out by her under my supervision for the last four and half years and to the best of my knowledge it has reached the standard, fulfilling the requirements of the regulations relating to the degree.

I further certify that the results contained in this thesis have not been submitted, either in part or in full, to any other university or Institute for the award of any Diploma or Degree.

Yash Paul

(YASH PAUL)
Department of Mathematics
Indian Institute of Technology
New Delhi - 110 016.

:ACKNOWLEDGEMENTS:

The results of my research and this thesis would not have been possible but for the able guidance and encouragement given by my supervisor Dr. Yash Paul to whom I am ever indebted to. I gratefully acknowledge facilities afforded by Professor O.P.Bhutani and Dr. M.M.Chawla, Head of the Department of Mathematics.

I can not also forget the cooperation and effective support given by my colleagues Mrs. A.Lalitha Dixit, Mr.P.Sreenivasa Rao and Mr. S.Gopalsamy during my tenure of research.

The main catalyst to the thesis and my research is the loving care extended by my parents and my ever accommodative husband, whose patience and assistance could not be described at all. I am grateful to them.

Finally, it is cutomary to acknowledge the services of the typist. But the entire thesis was typed and processed in our home computer - thanks to BBC Micro.

Sharadha

SHARADHA

:CONTENTS:

:ABSTRACT:

This thesis entitled "ON CERTAIN NON-ASSOCIATIVE RINGS AND ALGEBRAS WITH COMMUTATORS IN THE NUCLEI" consists of five chapters. Chapter I gives a brief review of the previous studies related to the present work. Chapter II deals with (γ,δ) rings ,in particular (-1,1) and (1,1) rings. More particularly, (-1,1) rings with commutators in the left nucleus has been studied. In the later part of this chapter (1,1) rings with commutators in the left nucleus and commutators in the middle nucleus have been studied. Here the left primitive rings and semi-simple rings are being taken for discussion. Using certain identities developed by I.R.Hentzel for (-1,1) rings, conditions on a maximal ideal $A \neq (0)$ are derived, under which R is associative. As a consequence a left primitive (-1,1) ring is shown to be associative.

The other main result in this section states that semi-simple (-1,1) rings are associative. Here the work of I.R.Hentzel has been freely used. Aside the above, it has been proved that a prime (-1,1) has no left ideal in the left nucleus.

The second part of the chapter has been devoted to prime (1,1) rings and idempotent $e \neq 0$. Certain identities involving the nucleus and the set of associators are given. The ideal generated by the subset (R,N) of commutators is

completely characterised and we have shown that it is contained in the nucleus. Using the Peirce decomposition we have derived the conditions under which an idempotent is an identity. Here a successful attempt has been made to prove that a prime (1,1) ring is associative.

In Chapter III prime alternative and prime right alternative rings with non-zero idempotent e are studied. We have given a complete characterisation of e with commutators in the nucleus. This chapter is divided into three sections. In the first section it has been proved that in the case of prime right alternative rings with commutators in the left nucleus, an idempotent $e \neq 0$, is an identity iff $e \in N$.

In the second section right alternative rings with commutators in the middle nucleus has been taken for elaboration. An ideal which is a subset of middle nucleus is constructed using a result proved in the earlier section. We arrived at the following: a prime ring is either associative or $N = Z$. The zero divisors in the middle nucleus are also studied.

In the last section using Peirce decomposition of R, we have proved that under certain conditions on R_{ij}, $i,j = 0,1$, R is associative. Behaviour of the subrings R_{ij}, $i,j = 0,1$ are also discussed completely.

In Chapter IV we have concentrated on finite-dimensional Antiflexible algebras. If H(a) is the ideal generated by the set (a,R,R), then it is shown that H(a) is associative and commutative, provided (H(a),a) = (0). Certain identities involving the elements of H(a), which were extensively used, in proving the other results of this chapter, are also given. It turns out that if (H(a),a) = (0) then under certain conditions a semi-prime antiflexible algebra is associative. As a corollary it follows that a nil semi-simple antiflexible algebras are also associative.

In the second section we have proved the following main theorem: If R satisfies the minimum conditions on right ideals then R satisfies a polynomial identity $A_n = (0)$ for some integer n, where A_1 is the ideal (R,R,R) + (R,R,R)R ...... and $A_n = (((A_{n-1}, R,R),R,R))$. Also we have proved that if H is a non-zero minimal right ideal contained in the middle nucleus then either $H^2 = (0)$ or it contains an idempotent.

In Chapter V, the last chapter of the thesis, using the structure of accessible rings given by E.Kleinfeld, behaviour of an idempotent e ≠ 0 has been studied. An attempt has been made to study Lattice ordered accessible rings, on the lines developed by M.C.Bhandari and A.Radhakrishna for antiflexible rings. Some sporadic results are given.