

**DESIGN AND DEVELOPMENT OF PREDICTIVE
CONTROL MODELS FOR A MULTI-FUEL RCCI ENGINE
USING EXPERIMENTAL PERFORMANCE DATA**

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INDIAN INSTITUTE OF TECHNOLOGY DELHI

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by

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Submitted

in fulfilment of the requirements of the degree of Doctor of Philosophy



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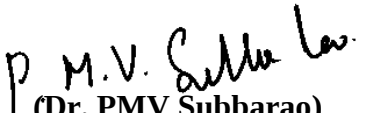
DEDICATED TO

MY PARENTS AND MY BELOVED WIFE

Certificate

This is to certify that the thesis entitled '**Design and Development of Predictive Control Models for a Multi-fuel RCCI Engine Using Experimental Performance Data**' submitted by **Mr. Chinmaya Mishra** to the Indian Institute of Technology Delhi, for the award of the degree of ***Doctor of Philosophy*** is a record of the original bonafide research work carried out by him under my guidance and supervision. The results contained in it have not been submitted in part or full to any other institute or university for the award of any degree/diploma.

Date: 26/10/2021


(Dr. PMV Subbarao)

Professor

Department of Mechanical Engineering

Indian Institute of Technology Delhi

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Doctoral research is a challenging expedition. It is a slow crucible of learning, interaction, exploration, wild ideas, mistakes, failures, pressure etc. with occasional taste of accomplishment and appreciation. A true research is all about continuous meddling, poking, and prying with problems, rather than relaxing with solutions as solutions are short-lived dwarves, whereas the problems are eternal titans.

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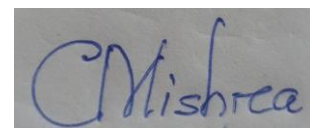
My heartiest gratitude also goes to my peers such as Dr. Pranab Das, Dr. Nikhil Kumar Singh, Dr. Abdul Rehman Khan, Mr. Devershi Maurya, Mr. Hariom Singh, Dr. L D Kala, Mr. Abhinav Chaterjee, Dr. Ved Praksh, Ms Daizy Rajput, Mr. Ankur Tiwari, Mr. Thochi Rengma, Mr. Puneet Valecha etc. These are wonderful boys and girls whose company I enjoyed a lot in terms of both overall learning, fun, and occasional fight.

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(Chinmaya Mishra)

Abstract

Reactivity controlled compression ignition (RCCI) engine is a dual fuel combustion technology to realize multi-order reduction in oxides of nitrogen (NO_x) and soot emissions, while achieving higher thermal efficiency. RCCI engine is based on a “unified engine” concept where the benefits of both high compression ratio stratified compression ignition operation and partial premixing can be simultaneously harnessed. RCCI combustion is realized by creating a partially premix and partially stratified in-cylinder mixture with fuels of opposing cetane rating or reactivity, as constituents. RCCI technology in standalone and hybrid mode can both complement and compete with the battery electric vehicles. Furthermore, RCCI technology also presents a suitable proposition to utilize various potential high and low cetane rating alternative fuels. In the present work, efforts were made towards four major directions in RCCI engine research.

Firstly, an attempt was made to retrofit a low-cost and mass use stationary diesel engine to operate in multi-fuel RCCI mode using gasoline, enriched biogas, and anhydrous ethanol as premix low cetane fuels (lcf) along with diesel as the common stratified high cetane fuel (hcf). Necessary instrumentation and combustion diagnostics were developed.

Secondly, experiments were conducted at various injection timings of diesel (θ_{soi}), low cetane fuels energy share (ES) and engine loads. The ES was restricted to 60% for gasoline and 70% for enriched biogas to avoid the need of exhaust gas recirculation (EGR). For ethanol, limited experiments were conducted up to 50% ES. The injection timings (θ_{soi}) were varied from 10°CA bTDC to 25°CA bTDC for gasoline-diesel RCCI. For enriched biogas-diesel RCCI, the same was little advanced between 12°CA bTDC to 27°CA bTDC to exploit the lower reactivity of the enriched biogas. The engine brake mean effective pressure (BMEP) sweep was between 0.953 bar to 4.271 bar. Experimental results suggested that increase in lcf energy share led to significant decline in NO_x and soot emissions for both

advanced and retarded θ_{soi} . However, for retarded θ_{soi} , high cyclic variations and rapid drop in combustion efficiency was witnessed, thereby restricting the upper limit of the lcf energy share. Furthermore, lower reactivity of lcf such as enriched biogas induced stronger cyclic variations at retarded θ_{soi} compared to gasoline. However, at advanced θ_{soi} , lower reactivity of the enriched biogas as lcf was found to be beneficial with no knocking symptoms. Similarly, at lower engine loads, enriched biogas induced more cyclic variations than gasoline in RCCI mode. The lower reactivity of the lcf was found to be an enabler towards load range extension in RCCI engines without the need of EGR. Experimental findings and literature indicated the need to carve out an optimal window for preferred RCCI benefit.

Thirdly, a genetic algorithm (GA) based multiobjective optimization was carried out. The experimental findings were used to develop high fidelity parametric maps using supervised learning based artificial neural networks (SNN). The chosen objective functions for the optimization were the emissions of NO_x , coefficient of variation of indicated mean effective pressure (COV_{IMEP}) and combustion efficiency (η_{comb}). The fidelity of the SNN parametric maps to capture experimental trends was found to be satisfactory. Based on the GA based multiobjective optimization, maximum energy share utilization and maximum pressure rise rate (MPRR) limits, the optimal θ_{soi} was chosen to be 20°C A bTDC for gasoline as lcf and 22°C A bTDC for enriched biogas as lcf in RCCI mode. For load sweep operations at fixed optimal θ_{soi} , suitable energy shares were also proposed.

Finally, the RCCI experimental data was used to design, develop and test of three independent mathematically evolved control models to address the RCCI engine “control of combustion” problem. The first control model involved a pure machine learning based black-box approach. The second control model was developed by integrating trained random forest (RF) machine learning and SNN to combustion physics-derived modified Livengood–Wu integral, parametrized double-Wiebe function, auto-ignition front

propagation speed based correlations, and residual gas fraction model. The third control model was developed by combining a stochastic pressure cycle reconstruction based tailored double-Wiebe function and RF machine learning to predict both the average and cyclic variation trends of combustion metrics. The last two control models were named as grey-box hybrid and quasi-white-box hybrid models, respectively. The objective of all the three control models were to predict the condition dependent combustion metrics, viz. peak pressure (PP), IMEP, MPRR, crank angles of occurrence for start of combustion (θ_{soc}), 50% mass fraction burnt (θ_{50}) and burn duration (θ_{BD}) across various RCCI operating regimes. A superior predictive capability of the proposed control models can provide a low-cost model driven control alternative against the expensive in-cylinder pressure sensor based feedback control in RCCI engines.

Results from the black-box model showed an error mean and standard deviation ($\mu_{err} \pm \sigma_{err}$) of -0.0465 ± 1.664 bar, 0.00056 ± 0.222 bar, -0.0001 ± 0.2263 bar/ $^{\circ}$ CA, $-0.0071 \pm 0.68^{\circ}$ CA, $-0.0217 \pm 1.07^{\circ}$ CA and $-0.0346 \pm 2.14^{\circ}$ CA for the predictions of PP, IMEP, MPRR, θ_{soc} , θ_{50} and θ_{BD} respectively for gasoline-diesel RCCI. For grey-box hybrid model the values were 0.3633 ± 1.76 bar, -0.0239 ± 0.22 bar, 0.0069 ± 0.27 bar/ $^{\circ}$ CA, $0.0554 \pm 0.71^{\circ}$ CA, $-0.0848 \pm 1.14^{\circ}$ CA and $0.0180 \pm 1.93^{\circ}$ CA in the same order. For quasi-white-box hybrid model, the values stood at 0.1283 ± 1.94 bar, -0.0004 ± 0.28 bar, 0.0043 ± 0.23 bar/ $^{\circ}$ CA, $0.0460 \pm 0.67^{\circ}$ CA, $0.0103 \pm 1.05^{\circ}$ CA and $0.0477 \pm 2.16^{\circ}$ CA for the aforementioned combustion metrics in the same order. The cyclic variation sensitivity of the quasi-white-box hybrid predictive model was found to be comparable with the black-box model. Similar trends were also observed for enriched biogas-diesel RCCI operations.

Keywords: Artificial neural network; auto-ignition front speed; combustion metrics; control models; cyclic variations; double-Wiebe function; energy share; genetic algorithm; Livengood–Wu integral ; Pareto front; random forest machine learning; RCCI etc.

सार

प्रतिक्रियाशीलता नियंत्रित संपीड़न प्रज्वलन (RCCI) इंजन उच्च तापीय दक्षता प्राप्त करते हुए नाइट्रोजन ऑक्साइड (NO_x) और कालिख उत्सर्जन में भारी कमी करने के लिए एक उन्नत तकनीक है। RCCI एक "एकीकृत इंजन" अवधारणा पर आधारित है जहां तापीय दक्षता लाभ के लिए स्तरीकृत संपीड़न प्रज्वलन के साथ उच्च संपीड़न अनुपात के साथ-साथ NO_x और कालिख उत्सर्जन में कमी के लिए आंशिक पूर्व-मिश्रण के लाभ प्राप्त किए जा सकते हैं। RCCI दहन को घटक के रूप में परस्पर विरोधी सिटेन रेटिंग या प्रतिक्रियाशीलता के ईंधन के साथ आंशिक पूर्व-मिश्रण और आंशिक स्तरीकृत इन-सिलेंडर मिश्रण बनाकर महसूस किया जाता है। स्टैंडअलोन और हाइब्रिड मोड में RCCI तकनीक बैटरी इलेक्ट्रिक वाहनों के पूरक और प्रतिस्पर्धा दोनों कर सकती है। इसके अलावा, RCCI प्रौद्योगिकी विभिन्न संभावित उच्च और निम्न सीटेन रेटिंग वैकल्पिक ईंधन का उपयोग करने के लिए उपयुक्त है। वर्तमान कार्य में RCCI इंजन अनुसंधान में चार प्रमुख दिशाओं की ओर प्रयास किया गया।

सबसे पहले, पूर्व-मिश्रण के लिए कम सिटेन ईंधन (एलसीएफ) के रूप में गैसोलीन, समृद्ध बायोगैस, और निर्जल इथेनॉल एवं स्तरीकृत संपीड़न प्रज्वलन के लिए डीज़ल को उच्च सीटेन ईंधन (एचसीएफ) के रूप में चुना गया। आवश्यक उपकरण और दहन निदान विकसित किए गए थे। एलसीएफ ऊर्जा हिस्सेदारी (ES), इंजन लोड और विभिन्न डीज़ल इंजेक्शन समय (θ_{soi}) पर प्रयोग किए गए। एग्जॉस्ट गैस रीसर्क्युलेशन (EGR) की आवश्यकता से बचने के लिए ES को गैसोलीन के लिए 60% और समृद्ध बायोगैस के लिए 70% तक सीमित रखा गया। इथेनॉल के लिए, 0% से 50% ईएस के लिए सीमित प्रयोग किए गए। गैसोलीन-डीज़ल RCCI के लिए θ_{soi} 10°C bTDC से 25°C bTDC तक भिन्न था। समृद्ध बायोगैस की कम प्रतिक्रियाशीलता का फायदा उठाया गया तथा θ_{soi} को 12 °C bTDC से 27 °C bTDC के बीच थोड़ा आगे किया गया। इंजन ब्रेक मीन इफेक्टिव प्रेशर (BMEP) 0.953 बार से 4.271 बार के बीच था। प्रायोगिक परिणामों ने सुझाव दिया कि एलसीएफ ऊर्जा हिस्सेदारी में वृद्धि से आगे और मंद दोनों θ_{soi} के लिए NO_x और कालिख उत्सर्जन में भारी गिरावट आई। हालांकि, मंद θ_{soi}

के लिए, उच्च चक्रीय भिन्नताएं और दहन दक्षता में तेजी से गिरावट देखी गई, जिससे एलसीएफ ऊर्जा हिस्से की ऊपरी सीमा सीमित हो गई। इसके अलावा, कम प्रतिक्रियाशीलता एलसीएफ जैसे समृद्ध बायोगैस ने गैसोलीन जैसे उच्च प्रतिक्रियाशीलता एलसीएफ की तुलना में मंद θ_{soi} पर उच्चतर चक्रीय भिन्नता का प्रदर्शन किया। हालांकि, डीजल θ_{soi} आगे होने पर समृद्ध बायोगैस की एलसीएफ के रूप में कम प्रतिक्रियाशीलता को बिना किसी असामान्य नॉकिंग के बगैर लाभकारी पाया गया। इसके विपरीत, गैसोलीन में हल्के नॉकिंग वाले लक्षण दिखाई दिए। इसी तरह, कम इंजन भार पर, समृद्ध बायोगैस ने RCCI मोड में गैसोलीन की तुलना में अधिक चक्रीय भिन्नताएं दिखाये। एलसीएफ की कम प्रतिक्रियाशीलता ईजीआर की आवश्यकता के बिना RCCI इंजनों में लोड रेंज विस्तार की दिशा में अनुकूल पाई गई। प्रायोगिक अध्ययनों ने अधिकतम RCCI लाभ के लिए एक इष्टतम ऑपरेशन विंडो तैयार करने की आवश्यकता का संकेत दिया। पैरामीट्रिक अनुकूलन के लिए एक आनुवंशिक एल्गोरिथम (जीए) आधारित बहुउद्देश्यीय अनुकूलन किया गया था।

प्रयोगात्मक निष्कर्षों का उपयोग पर्यवेक्षित शिक्षण आधारित कृत्रिम तंत्रिका नेटवर्क (एसएनएन) का उपयोग करके उच्च निष्ठा पैरामीट्रिक मानचित्र विकसित करने के लिए किया गया। अनुकूलन के उद्देश्य कार्य NO_x , संकेतित माध्य प्रभावी दबाव (COV_{IMEP}) और दहन दक्षता (η_{comb}) की भिन्नता का गुणांक थे। प्रयोगात्मक प्रवृत्तियों को पकड़ने के लिए एसएनएन पैरामीट्रिक मानचित्रों की निष्ठा संतोषजनक पाई गई। जीए आधारित बहुउद्देश्यीय अनुकूलन, अधिकतम ऊर्जा हिस्सेदारी उपयोग और अधिकतम दबाव वृद्धि दर सीमाओं के आधार पर, इष्टतम θ_{soi} गैसोलीन के लिए एलसीएफ के रूप में $20^\circ CA$ bTDC और समृद्ध बायोगैस के लिए $22^\circ CA$ bTDC चुना गया। RCCI मोड में एलसीएफ नियत इष्टतम θ_{soi} पर लोड संचालन के लिए, उपयुक्त ऊर्जा शेयरों का भी प्रस्ताव किया गया। अंत में, RCCI प्रयोगात्मक डेटा का उपयोग दहन समस्या के RCCI इंजन नियंत्रण को संबोधित करने के लिए तीन स्वतंत्र गणितीय रूप से विकसित नियंत्रण मॉडल के डिजाइन, विकास और परीक्षण के लिए किया गया। पहले नियंत्रण मॉडल में एक शुद्ध मशीन लर्निंग आधारित ब्लैक-बॉक्स दृष्टिकोण

शामिल था। दूसरा नियंत्रण मॉडल प्रशिक्षित यादृच्छिक वन (आरएफ) मशीन लर्निंग और एसएनएन को दहन भौतिकी-व्युत्पन्न संशोधित लिवेनगूड-वू इंटीग्रल, पैरामीट्रिज्ड डबल-विबे फ़ंक्शन, ऑटोइग्निशन फ्रंट प्रोपेगेशन स्पीड आधारित सहसंबंध, और अवशिष्ट गैस अंश मॉडल को एकीकृत करके विकसित किया गया। तीसरा नियंत्रण मॉडल दहन मेट्रिक्स के औसत और चक्रीय भिन्नता प्रवृत्तियों दोनों की भविष्यवाणी करने के लिए एक अनुरूप डबल वीबे फ़ंक्शन (डी-डब्ल्यू) और आरएफ मशीन लर्निंग को मिलाकर विकसित किया गया। पिछले दो नियंत्रण मॉडल को क्रमशः ग्रे-बॉक्स हाइब्रिड और अर्ध-सफेद-बॉक्स हाइब्रिड मॉडल के रूप में नामित किया गया। सभी तीन नियंत्रण मॉडल का प्राथमिक उद्देश्य चयनित स्थिति पर निर्भर इन-सिलेंडर दहन मेट्रिक्स अर्थात्, पीक प्रेशर (पीपी), आईएमईपी, एमपीआरआर, दहन की शुरुआत के लिए क्रैंक एंगल्स (θ_{soc}), 50% मास फ़ैक्शन बर्न (θ_{50}) और बर्न अवधि (θ_{BD}) आदि की भविष्यवाणी करना था। नियंत्रण मॉडल की एक बेहतर भविष्य कहनेवाला क्षमता RCCI इंजनों में महंगे इन-सिलेंडर सेंसर आधारित फीडबैक नियंत्रण के खिलाफ कम लागत वाला मॉडल संचालित नियंत्रण विकल्प प्रदान कर सकती है।

ब्लैक-बॉक्स मॉडल के परिणामों में -0.0465 ± 1.664 बार, 0.00056 ± 0.222 बार, -0.0001 ± 0.2263 बार/°सीए, -0.0071 ± 0.68 °सीए, -0.0217 ± 1.07 °सीए और -0.0346 ± 2.14 °सीए का त्रुटि माध्य और मानक विचलन क्रमशः पीपी, आईएमईपी, एमपीआरआर, θ_{soc} , θ_{50} और θ_{BD} की भविष्यवाणियों के लिए गैसोलीन- डीजल RCCI में पाया गया। ग्रे-बॉक्स हाइब्रिड मॉडल के लिए मान 0.3633 ± 1.76 बार, -0.0239 ± 0.22 बार, 0.0069 ± 0.27 बार/° सीए, 0.0554 ± 0.71 °सीए, -0.0848 ± 1.14 °सीए और 0.0180 ± 1.93 °सीए इसी क्रम में थे। अर्ध-सफेद-बॉक्स हाइब्रिड मॉडल के लिए, मान 0.1283 ± 1.94 बार, -0.0004 ± 0.28 बार, 0.0043 ± 0.23 बार/°सीए, 0.0460 ± 0.67 °सीए, 0.0103 ± 1.05 °सीए और 0.0477 ± 2.16 °सीए पर था। उसी क्रम में उपरोक्त दहन मेट्रिक्स। अर्ध-सफेद-बॉक्स हाइब्रिड भविष्य कहनेवाला मॉडल की चक्रीय भिन्नता संवेदनशीलता ब्लैक-बॉक्स मॉडल के साथ तुलनीय पाई गई। समृद्ध बायोगैस-डीजल RCCI संचालन के लिए भी इसी तरह के रुझान देखे गए।

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Acronyms

A	Piston bore area in m ²
AI	Artificial intelligence
AKI	Anti-knock index
A.RCCI	Aggressive RCCI optimal
aTDC	After TDC
B	Number of trees in a random forest predictor
BMEP	Brake mean effective pressure
BP	Brake power
bTDC	Before TDC
CAI	Controlled auto-ignition
CDC	Conventional diesel combustion
CI	Compression ignition
CN _{mix}	Mixture cetane rating
CO	Carbon monoxide
CO ₂	Carbon dioxide
COM	Control-oriented models
COV	Coefficient of variation
C.RCCI	Conservative RCCI optimal
CRDI	Common rail direct injection
CSV	Comma separated values
DI	Direct injection
DTBP	Di-tert-butyl peroxide
E85	15% Ethanol and 85% gasoline blend
EBES	Enriched biogas energy share
ECU	Electronic control unit
EES	Ethanol energy share
EGR	Exhaust gas recirculation
ES	Energy share
FSN	Filter smoke number
GA	Genetic algorithm
GDI	Gasoline direct injection
GES	Gasoline energy share
H ₂	Hydrogen
H ₂ S	Hydrogen sulfide
HC	Hydrocarbons (unburnt)
HCCI	Homogeneous charge compression ignition
hcf/lcf	High and low cetane (rating) fuels
HPCC	Highly premixed charge combustion
HRR	Heat release rate
IMEP	Indicated mean effective pressure
ISFC	Indicated specific fuel consumption
kVA	Kilovolt ampere
LCV	Lower calorific value
LPV	Linear parameter varying
MFB	Mass fraction burnt
MKIM	Modified knock integral method

MPa	Mega Pascal
MPC	Model predictive controller
MPC563	Microcontroller
MPRR	Maximum pressure rise rate
N	Engine speed in rev/min
N_{RF}	Number of trees in the random forest
N_2	Nitrogen
NH_3	Ammonia
OH	Hydroxyl radical
PAH	Polycyclic aromatic hydrocarbon
PDF	Probability density function
PFI	Port fuel injector
PI	Proportional-integral controller
PLTC	Lean premixed and low temperature combustion
PODE	Polyoxymethylene dimethyl ethers
PP	Peak cylinder pressure
ppm	Parts per million
PPR	Pulses per rotation
PRF	Primary reference fuel
PRR	Pressure rise rate
RCCI	Reactivity controlled compression ignition
RON	Research octane number
SI	Spark ignition
sMAPE	Symmetric mean absolute percentage error
SMPS	Switched mode power supply
SNN	Supervised learning based artificial neural networks
SVM	Support vector machines
TDC	Top dead center
ULSD	Ultra-low sulfur diesel
WN	Wobbe number
WP_{dw}	Double-Wiebe parameter vector
WP_{mdw}	Double-Wiebe parameter matrix

Nomenclature

General Nomenclature

A_{orifice}	Cross sectional area of the orifice
a	D-W efficiency factor
Ar	Argon
A_{wall}	Variable wall area
C	Piezoelectric sensor time constant
C_d	Coefficient of discharge
CH_4	Methane
C_{loss}	Experimental indicated thermal efficiency to air standard efficiency ratio
COV_{IMEP}	Coefficient of variation of indicated mean effective pressure
C_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
c_1 - c_{20}	Coefficients for coupling D-W parameters with engine operating condition
C_2H_2	Acetylene
C_2H_4	Ethylene
C_3H_6	Propene
C_6H_6	Benzene
c_1 - c_{20}	Coefficients for coupling D-W parameters with engine operating condition
D	Mass diffusivity
Da	Damköhler number
D_1/D_2	Intake and exhaust valve diameter
D-W	Double Wiebe function
E_i/M_i	Cycle resolved experimental/model predictions of the combustion metrics
F/A	Fuel-air ratio
F_1/F_2	First and second D-W amplitude correction factor
h	Convective heat transfer coefficient
h_{water}	Water column height
I_{MKIM}	Modified knock integral
I_{LW}	Livengood-Wu integral
k	Total data samples
Kurt	Kurtosis
L	Engine stroke length in meter
L_1/L_2	Intake and exhaust valve lift
m_a	Mass flow rate of the air
m_{bf}	Backflow residuals
m_{trap}	Trapped residuals
m_f	Mass flow rate of the fuel
m_1/m_2	First and second D-W shape factors
N	Engine speed in rev/min
Nm^3/h	Normal cubic meters per hour
NO_x	Oxides of nitrogen
n	Polytropic index
O_2	Oxygen
P, V, T, R_{ex}	Pressure, volume, temperature, characteristic gas constant for exhaust gases
P_{abs}	Absolute in-cylinder pressure
P_{atm}	Atmospheric pressure

pC	Pico-coulomb
P_{drum}	Air-box drum pressure
P_d/P_u	Downstream and upstream pressure
P_{im}	Intake manifold pressure
P_m	Motoring pressure
P_r, V_r, T_r	Reference pressure, volume and temperature
P_r	Raw pressure
P_s	Smooth pressure
Q_{D-W}	D-W heat release
q_{ex}	Specific energy
Q_1/Q_2	First and second D-W net heat input
Re	Reynolds number
R_g	Characteristic gas constant
R^2	Coefficient of determination of linear regression
r_{ex}	Blow-down to post-blow-down heat loss ratio
rpm	Rev/min
Sa	Sankaran number
$S_{\text{ign}}/S_{\text{dflg}}$	Auto-ignition/deflagration front propagation speed
Sk	Skewness
T	Temperature
t	Time relapse
T_{ex}	Exhaust temperature
T_{im}	Intake manifold temperature
T_{rg}	Residual gas temperature
Theil U II	Uncertainty metric
t_{ign}	Time instant at which ignition begins
U_m	Mean piston speed
v/v	Volumetric concentration
v_c	Ratio between volume at any crank angle $V(\theta)$ to $V_{\theta_{\text{ivc}}}$
V_s	Swept volume
W	Fuel tank weight
WPC	Work done per cycle
x_{bdw}	Double-Wiebe burn rate
X_{rg}	Residual gas fraction

Greek symbols

θ_{soc}	Crank angle of occurrence for start of combustion
θ_{soi}	Start of injection crank angle
ρ_{air}	Density of air in kg/m^3
ρ_{water}	Density of water in kg/m^3
$^{\circ}\text{C}$	Degree Celsius
η_{comb}	Combustion efficiency
Φ_{overall}	Overall equivalence ratio
Φ_{st}	Stoichiometric equivalence ratio
$dQ_{\text{net}}/d\theta$	Net heat release rate
θ_{ivc}	Intake valve closing crank angle
χ	Mole fraction
A_{δ}	Piston passage area
$dQ_{\text{whl}}/d\theta$	Wall heat loss rate

θ_{evo}	Exhaust valve opening crank angle
θ_{evc}	Exhaust valve closing crank angle
θ_{ref}	Reference crank angle during exhaust when cylinder pressure is 1 bar
μ	Arithmetic mean
$\sigma_{68.3}$	Standard deviation with 68.3% confidence level in a normal distribution
$\sigma_{99.7}$	Standard deviation with 99.7% confidence level in a normal distribution
η_{RER}	Realistic efficiency ratio
η_{AirstdSI}	Air standard efficiency in SI mode
η_{AirstdCI}	Air standard efficiency in CI mode
η_{ITE}	Indicated thermal efficiency
τ_1/τ_2	Ignition delays for low cetane/high and low cetane fuel-air mixture
$\alpha_1\text{-}\alpha_{10}$	MKIM model coefficients
θ_1/θ_2	First and second D-W burn duration
β	First phase fuel mass fraction burnt
ψ	Fraction of low cetane fuel burnt in first phase
$c_1\text{-}c_{20}$	Coefficients for coupling D-W parameters with engine operating condition
$\lambda_1\text{-}\lambda_{10}$	Auto-ignition front model coefficients
α	Blow-by volumetric efficiency compensation
μ_{err}	Mean of error between model prediction and experimental data
σ_{err}	Standard deviation of error between model prediction and experimental data
θ_{50}	Crank angle of occurrence for 50% fuel mass fraction burnt
θ_{10}	Crank angle of occurrence for 10% fuel mass fraction burnt
θ_{90}	Crank angle of occurrence for 90% fuel mass fraction burnt
θ_{BD}	θ_{10} to θ_{90} burn duration in crank angle degrees